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**INTEREST RATE  
TERM STRUCTURE IN LATVIA  
IN THE MONETARY POLICY CONTEXT**

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## ABSTRACT

This paper examines applicability of various models of the yield curve construction to the Latvian money and government securities markets, and analyses the information content implied in the yield curve. The rejection of hypothesis about the existence of a zero risk premium leads to an inference that forward rates in general do not ensure unbiased forecasts of spot rates, and the pure interest rate expectations theory cannot be applied in interest rate forecasting. Long-term interest rates contain a risk premium that is other than zero. This conforms well with the results obtained from studies conducted on the financial markets of developed countries.

**Key words:** *term structure of interest rates, risk premium, the Nelson-Siegel model*

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The views expressed in this publication are those of the author (Head of the Financial Market Analysis Division, Monetary Policy Department of the Bank of Latvia, at the time of making the study). The author assumes responsibility for any errors or omissions.

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## CONTENTS

Introduction	4
1 Theoretical Principles of the Interest Rate Term Structure	6
1.1 Yield in the Context of the Interest Rate Term Structure	6
1.2 Role of the Interest Rate Term Structure in Monetary Policy	9
2 Modeling of the Interest Rate Term Structure	13
2.1 Methods of Modeling the Interest Rate Term Structure	13
2.2 Yield Curve Modeling Experience of Central Banks of Other Countries	18
2.3 Application of the Interest Rate Term Structure Modeling Methods to Processing of Latvia's Data	19
3 The Interest Rate Expectations Theory	28
3.1 Essence of the Interest Rate Expectations Theory and Basic Principles of Its Testing	28
3.2 Testing of the Interest Rate Expectations Theory on the Latvian Financial Market	31
4 Application of the Interest Rate Term Structure to the Bank of Latvia's Monetary Policy	36
Conclusions	49
Appendices	51
Bibliography	60

## INTRODUCTION

Since the 1990s, national central banks, when facing a more complex monetary transmission process, have been focusing greater attention on the pricing of various financial assets and the information implied in them. There is an assumption that the pricing of assets expands the range of variables available to central banks, and hence enriches the information amassed by them through assessing conventional indicators (e.g. monetary aggregates and real GDP growth factors). The analysis of the term structure of interest rates or the yield curve, which has become a standard instrument in the monetary policy decision-making in developed countries, has a significant role in this process.

Data implied in the term structure of interest rates for central banks figure as a valuable source of supplementary information on market forecasts in respect of a number of fundamental macroeconomic variables, including expected changes in short-term interest rates. The term structure of interest rates is used in obtaining information on financial market expectations and as an indicator preceding the business cycle.

Owing to underdeveloped financial markets, however, the application of the yield curve for the above-referred purposes has, until recently, been rather limited in the developing countries, Latvia among them. But due consideration of quite an impressive recent development of the Latvian securities and money markets, and the brisk trading on the secondary market lead to an assumption that the application to the Latvian financial market of technical approaches reviewed in the academic literature and practically tested in financial markets of developed countries would seem quite logical.

For the Bank of Latvia, the importance of identification and assessment of information implied in the prices of financial assets will increase in the near future in line with Latvia's integration with the European Union (EU) and the Economic and Monetary Union (EMU). Upon Latvia entering the common economic and monetary space, the Bank of Latvia will have to re-direct its research activities in pursuit of such monetary policy goals that are common for the euro area. In the process of assessing economic effects of the monetary policy and defining its strategy, the European Central Bank (ECB) relies upon the assessment of a wide range of financial factors and indicators. Consequently, when the Bank of Latvia joins the Eurosystem, it will have to make a regular research contribution to identification and assessment of financial indicators like other central banks.

This paper examines applicability of various models of the yield curve construction to the Latvian money and government securities markets, and analyses the information content implied in the yield curve.

Chapter 1 sums up theoretical principles for the interest rate term structure and gives an overview of the yield curve application to contemporary monetary policy. Chapter

2 is devoted to the methodology of building the present zero-coupon (hereinafter, spot) and the future expected (hereinafter, forward) yield curves by means of different types of modeling. Chapter 3 deals with the interest rate expectations theory. Chapter 4 provides analysis of the results obtained in relation to the Bank of Latvia's monetary policy. The Conclusions summarises core outcomes of the paper.

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# 1 THEORETICAL PRINCIPLES OF THE INTEREST RATE TERM STRUCTURE

## 1.1 Yield in the Context of the Interest Rate Term Structure

The term structure of interest rates reflects the dependence of interest rates on maturity. The yield curve is a graphical expression of this dependence. It is essential to note that interest rates that are situated on the yield curve have identical risk and liquidity levels as well as the same tax conditions. In order to construct a yield curve, market yields of certain financial instruments with various maturities are usually used.

In practice, yield curves are usually constructed using yield data of a special category of securities with the lowest risk and the broadest spectrum of instruments in terms of their number and maturity range, namely the government debt securities, hence a closer focus on the yield of this type of financial instruments.

The market price of a bond is known to be the value of all future cash flows in relation to this particular bond.

$$P = \frac{C}{1 + y(m)} + \frac{C}{1 + (y(m))^2} + \dots + \frac{C + R}{(1 + y(m))^m} \quad [1.1],$$

where  $P$  is the price of the bond;

$C$  is the coupon of the bond;

$R$  is the face value of the bond;

$y(m)$  is the yield to maturity of the bond maturing in period  $m$ .

In comparison with developed countries where bonds have already been quoted for the past century and a half, bonds on the Latvian securities market is a new financial instrument. The Government of Latvia first issued securities in 1993. Currently, most widely spread securities in Latvia are government bonds with the maturity of 5 and 10 years, and they are coupon bonds.

One of theoretical deficiencies of the yield curve  $y(m)$  is related to the reinvestment risk. The given method for calculating a bond price actually implies that all up-coming coupon payments will be reinvested at one and the same rate, which seldom materialises in practice. This deficiency is offset by using the spot yields and the yield curve, their graphic presentation. The spot interest rate is an interest rate at which each separate cash flow of a bond is discounted (coupons and the face value). The spot interest rate is often called zero-coupon, as it shows the yield-to-maturity of a hypothetic discount bond or a zero-coupon bond. If spot interest rates are known, the market price of a bond will be as follows:

$$P = \frac{C}{(1 + {}_0y_1)} + \frac{C}{(1 + {}_0y_2)^2} + \dots + \frac{C + R}{(1 + {}_0y_m)^m} \quad [1.2],$$

where  ${}_0y_m$  is the spot interest rate of the cash flow maturing at the end of period  $m$ .

In order to arrive at an accurate price of a coupon security, market participants must always know the spot interest rate. A more complicated calculation of these rates in comparison with the yield-to-maturity  $y(m)$  is the main weakness. As direct market spot interest rates are typical only for short-term discount securities (without coupon payments), they must be estimated using available market prices of coupon bonds.

Equation [1.2] is often written using discount factors:

$$P = \delta_1 C + \delta_2 C + \dots + \delta_M (C + R) \quad [1.3],$$

where  $\delta_m$  is the discount factor of period  $m$  ( $m = 1, \dots, M$ ) and transformation of the spot interest rate of period  $m$ :

$$\delta_m = \frac{1}{(1 + {}_0y_m)^m} \quad m = 1, \dots, M \quad [1.4].$$

The discount curve always is downward sloping, and the discount value at the time moment ( $m = 0$ ) is always equal to 1, i.e.  $\delta_0 = 1$ . This assumption is a serious restriction indicating that the amount of cash received at the present moment is not discounted.

The corresponding spot interest rate is:

$${}_0y_m = \left( \frac{1}{\delta_m} \right)^{1/m} - 1 = \sqrt[m]{\frac{1}{\delta_m}} - 1 \quad [1.5].$$

It is the spot interest rate that characterises the term structure of interest rates.

In respect of the yield curve, continuous variables are more often used in the financial theory. For example, in contrast to the discrete function, the continuous discount function reflects dependence of the discount factor on the maturity for any moment. In fact, the continuous discount function  $d_m$  is the present value of one cash unit receivable in period  $m$ . Further discussion in this paper shall be based on the use of continuous variables.

The price of a bond in the event of a continuous discount function shall be:

$$P = C \sum_{m=1}^M d_m + d_M R \quad [1.6],$$

where  $d_m = e^{-{}_0r_m m}$ ,  
in which  ${}_0r_m$  is the continuous spot rate.



Correspondingly, the spot rate function is derived from the respective discount function in the following way:

$$-{}_0r_m m = \ln(d_m) \Rightarrow {}_0r_m = -\frac{\ln(d_m)}{m} \quad [1.7].$$

Some re-arrangement of equation [1.6] is required in line with the coupon payment dates. Although the calculation of compounded income is simple, it may present an inconvenience for empirical studies. To escape such an inconvenience, it is assumed that the coupon payments are made continuously (not on fixed dates), and no compounding occurs. The assumption regarding continuous calculation of income indicates that the bond price equation can be simplified:

$$P = C \int_0^M d(\mu) d\mu + R d_M \quad [1.8].$$

Implied information plays an important role in the theory of the term structure of interest rates. Though not visible at a first glance, it is incorporated in the price of a financial asset. The implied forward rate curve and its shifts are used most often. The forward rate curve is derived from either the discount function or from the spot rate function. The use of the spot rate curve in the production of the forward rate curve appears to be more convenient:

$$e^{-{}_0r_m M} = e^{-{}_0r_m m} e^{-{}_m f_M (M - m)} \quad [1.9],$$

$$-{}_0r_m M = (-{}_0r_m m) + (-{}_m f_M)(M - m) \quad [1.10],$$

$${}_m f_M = \frac{{}_0r_m M - {}_0r_m m}{M - m} \quad [1.11],$$

where  ${}_m f_M$  is the forward rate to be applicable to a future loan extended in period  $m$  and repayable in period  $M$ .

The spot interest rate  ${}_0r_m$  can be taken for the geometrical mean of all the implied forward rates.

In order to carry a valuable informative content about the future interest rates, there are fundamental preconditions for a yield curve to meet. One of such conditions is related to the interest rate expectations theory, which stipulates that short-term forward rates incorporated in today's long-term interest rates are closely related to market expectations for actual short-term interest rates in the future. There are two types of the interest rate expectations theory: the pure interest rate expectations theory and the interest rate expectations theory.

The interest rate expectations theory in its pure form is based on an assumption that

forward rates only reflect the interest rates expected in the future. Thus, the yield on the long-term bonds is equal to the average of the expected yield on the short-term bonds:

$${}_t r_{t+M} = \frac{1}{k} \sum_{i=0}^{k-1} E_t ({}_{t+mi} r_{t+m(i+1)}) \quad k = M/m \quad [1.12],$$

where  ${}_t r_{t+M}$  is the long-term ( $M$ ) spot rate in period  $t$ ;  
 $E_t$  is the expectations operation sign;  
 ${}_{t+mi} r_{t+m(i+1)}$  is the short-term ( $m$ ) spot rate in period  $t$ .

The expectations theory, in turn, allows for existence of risk premiums for maturities, but argues that it is constant for a definite maturity and the same for bonds with different redemption dates. Under this formulation, the interest rate expectations theory (often defined also as the theory of rational expectations in respect of the interest rate term structure) is incorporated in the majority of financial theories and macroeconomic textbooks.

## 1.2 Role of the Interest Rate Term Structure in Monetary Policy

Information contained in the term structure of interest rates furnishes central banks with valuable additional data regarding market expectations for a number of significant macroeconomic variables, including also future changes in short-term interest rates.

One of the reasons underlying the focus of the central banks on this type of information is the growing uncertainty about operation of the monetary transmission mechanism and, in particular, non-sustainability of the demand for money in the mid-1980s, which impaired credibility of interim goals of monetary policy's simple monetary aggregates in the majority of countries. From this perspective, various financial market indicators can potentially provide information about on-going changes also in the periods of time when monetary transmission is not visible, since financial markets adjust faster to changes in the situation, including also those in monetary policy, than goods markets. In addition, financial asset prices reflect market expectations because they are inherently forward-looking, i.e. their prices depend on future processes on financial markets and economy as a whole as opposed to goods and labor markets where past processes apparently have a more notable effect on prices.

Despite growing difficulties regarding the treatment of changes in monetary aggregates in many countries, they still are used as fundamental economic indicators by many central banks (27); given their relatively strong structural linkage with the ultimate target of monetary policy, monetary aggregates are believed to forecast the real economic development and inflation better than the financial market indicators. Furthermore, many central banks strongly rely on the relationship between monetary aggregates and the real economy, as the theoretical foundation of this relationship

emerged long ago. Nevertheless, it should be noted that the estimation of financial asset prices is acquiring an ever-growing importance for monetary policy of a number of central banks, particularly those whose primary objective is inflation targeting. For instance, financial asset prices as indicators play an important role in the second pillar in the context of the Eurosystem's two-pillar monetary policy strategy.

Implied information gives an opportunity to verify conventional indicators and obtain additional data about direct causes or initial signals of inflation, all enriching assumptions derived from other models. For instance, if monetary policy analysts arrive at forecasts that are supported by signals obtained earlier from the interest rate term structure, such forecasts will be more credible. Provided the interest rate term structure gives signals opposite to those obtained through conventional channels, analysts are likely to check future forecasts more carefully.

Traditionally, the slope of the yield curve is considered to be a useful indicator for the assessment of future economic activity forecasts. An ascending yield curve is usually perceived as signaling an increase in economic activity, while a smoothing or a downward-sloping term structure is a sign of an anticipated decline in the growth. There may be several causes for such an interrelation between the shape of the yield curve and future growth forecasts of the real sector. Thus, for instance, if financial market participants expect a decline in economic growth in the future, they would seek to push up the demand for long-term bonds in an attempt to use bond yields in offsetting the loss of income due to deteriorating economic situation. Consequently, the price of bonds will increase in comparison with the price of short-term debt instruments, whereas the income from bonds will decrease in comparison with the income from short term debt instruments. This will result in the smoothing of the yield curve.

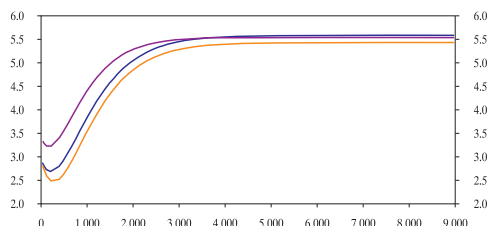
Chart 1.1 shows short-term forward rates in the euro area at different dates in 2002 and 2003. The positioning of the forward rate curves differs. The ECB admits that the different positions of the yield curves indicate that in the given period of time market forecasts concerning future short-term interest rate movements in the euro area over both shorter and longer horizons have changed substantially, and namely, a reduction in interest rates has been observed.(16) A drop in interest rates has been influenced by a shift of resources from stock markets to more risk-averse opportunities of invest-

Chart 1.1

**IMPLIED SHORT-TERM FORWARD RATES OF THE EURO**

(curves are constructed on the basis of LIBOR EUR data obtained from Reuters, using the Svensson model; in days; %)

- 10.09.2002
- 10.01.2003
- 10.02.2003



ing – the government securities markets, as is typical for economically unfavorable periods. However, the primary reason for a decrease in yields on government bonds is the growing pessimism among market participants about the economic development in the euro area in the short and medium term, based on disseminated discouraging economic data, higher prices for oil and growing geopolitical instability. By contrast, the short-term forward rates have changed little over a long horizon, indicating that market participants have not developed a different assessment of the long-term economic growth in countries of the euro area.

It should be noted, however, that the results obtained by different scholars on the relationship between the yield curve and macroeconomic indicators are often contradictory. For instance, studies by A. Estrella and F. Mishkin (19) indicate that the yield curve is a powerful instrument in forecasting real growth and inflation in the EMU for one to two upcoming years. J. Berk and P. Bergeijk (7), on the other hand, arrived at quite opposite results and argued that practical application of the yield curve in forecasting inflation and the real growth in the euro area is of limited nature.

Implied information is of great tactical importance over a short horizon. One of the most complicated questions in monetary policy is as follows: when and how much should a central bank increase or reduce the short-term interest rate to reach (or maintain) the desirable inflation target? In practice, a central bank usually increases or cuts interest rates gradually to follow the effects of the changes and to probe the necessity for further such changes. Monetary policymakers know that it takes several months (probably even years) for the interest rate shifts – increases or cuts – to be felt in the economy. That is why a central bank needs financial indicators that would enable it to check the effects of the measures implemented and the response of the economy to them.

For the purpose of analysis, central banks often choose such a maturity of forward rates, which is equivalent to the central bank's interest rate instrument (e.g. repo transactions). In this case, the forward rate reflects market participants' forecasts regarding the interest rate policy of the central bank. It is important to note that as long as an increase in interest rates is included in market participants' forecasts it does not indicate new signals of the central bank's monetary policy. Should the market forecast be opposite to the assessment of the central bank, the latter would strive not to shock market participants with its activity.

As has already been stated, various financial instruments, not necessarily only traditional government bonds, can be used in estimating the yield curve. In recent decades, market participants have heavily relied on government securities' yield curves as a benchmark for determining the value of attracted resources at different maturities. However, payment obligations of the private sector, particularly bonds secured by collateral and interest rate swaps (hereinafter in this Chapter, swaps) can also be used in constructing benchmark yield curves.

Quite recently, interest rates of government securities were synonyms to risk-free interest rates; the yield curve of the benchmark interest rate used in identifying market expectations, however, does not need to be a risk-free yield curve. The absence of a risk premium is not a precondition for identifying market participants' expectations. To arrive at market forecasts for macroeconomic factors, risk premiums incorporated in forward rates must be predictable.

Today market participants more often rely on the fixed interest rate used in swaps as a benchmark for return. The swift growth of swap market in the late 1990s was associated with the reduction of liquidity premium incorporated in interest rates and strengthening of the market. Debt commitments of almost all banks are based on the short-term interbank interest rate, e.g. LIBOR and EURIBOR. That explains propensity of banks to set prices in accord with the fixed interest rate curve of swaps in which forecasts of the future LIBOR or EURIBOR interest rates are implied. As securities markets of different countries are not homogeneous, securities yield curves of these countries are difficult to compare. By contrast, using swaps is a relatively easy way of comparing yields on different financial instruments across different countries. Governments have even started using swaps in the risk management of potential losses. Transition to swaps has become most popular on the euro market, as investors quickly became aware of advantages offered by the yield curve of the euro swaps being used as a benchmark.

In the process of assessing interest rate forecasts of the market participants' short-term operations, the ECB makes use of two important reference rates of unsecured (transactions without a collateral) market – the EONIA, the euro overnight index average, and the EURIBOR, the euro interbank offered rate. Together they provide references to interest rates on loans starting with overnight and finishing with loans maturing in one year.

Though a yield curve can be constructed basing on prices of various financial instruments, this study will focus only on such fixed asset prices that are based on fixed income securities and money market instruments. An appropriate reflection of the term structure of interest rates is the foundation on which studies conducted on the interest rate expectations theory rest. In line with it, the next Chapter deals with theoretical aspects of the methodology for yield curve modeling enabling the construction of spot and forward rate curves.

## 2 MODELING OF THE INTEREST RATE TERM STRUCTURE

### 2.1 Methods of Modeling the Interest Rate Term Structure

Computation of spot and forward rates is straightforward when the spot yields and also the discount function are simultaneously available for transactions with different terms to maturity. Given that very often, however, prices of financial instruments are not available for all maturities, other specific estimation methods need to be applied to obtain the discount function using prices of financial instruments available on the market.

The purpose of the yield curve modeling is to produce a possibly smooth function using existing market rates, which would reflect the actual data most accurately and, at the same time, be instrumental for further study of market expectations.

When modeling the yield curve, three basic groups of methods are used: polynomials and splines, the stochastic factor and the general equilibrium methods. The methods of polynomials and splines are based on smoothing of the interest rate term structure without dwelling upon factors that influence it. The stochastic factor and general equilibrium models construct the yield curve basing on the factors with an effect on it. This study deals with the following yield curve modeling methods for the construction of Latvia's yield curve: the polynomial method, the spline method, stochastic factor methods (Vasicek, Nelson–Siegel and Svensson models) and the general equilibrium methods (Cox–Ingersoll–Ross model).

#### *Polynomial and Spline Methods*

In polynomial modeling, it is assumed that the discount function  $d_m$  can be described as a sum of  $k$  basis functions  $f_j(m)$  ( $j = 1, \dots, k$ ) (plus 1), whose coefficients ( $a_j, j = 1, \dots, k$ ) are estimated:

$$d_m = 1 + \sum_{j=1}^k a_j f_j(m) \quad [2.1],$$

where  $f_j(m)$  is the  $j$  basis function;  
 $a_j$  is the coefficient of the  $j$  basis function.

Selection of the basis function  $f_j(m)$  is the decisive factor. Where equation [2.1] uses a simple polynomial, the discount function is determined by a  $k$ -degree polynomial:

$$d_m = 1 + a_1 m + a_2 m^2 + \dots + a_k m^k \quad [2.2].$$

Thus, for example, selecting  $m, m^2, m^3$  as basis functions ( $f_1(m) = m, f_2(m) = m^2, f_3(m) = m^3$ ), the price of the bond  $i$  with two payments  $c_1$  and  $c_2$  in time moments  $m_1$  and  $m_2$  is calculated as follows:

$$(P_{pol})_i = c_1(1 + a_1m_1 + a_2m_1^2 + a_3m_1^3) + c_2(1 + a_1m_2 + a_2m_2^2 + a_3m_2^3) \quad [2.3].$$

In search for parameters of the discount function, bond quotations on the market, i.e. bond prices valid on the market at a particular moment, are used. The discount function is written basing on the principles of the least square method (LSM), i.e. theoretical bond prices must maximally correspond to actually valid bond market prices:

$$\sum_{i=1}^N (P_i - (P_{pol})_i)^2 \rightarrow \min \quad [2.4].$$

Simple polynomials are subject to criticism mainly because of the trade-off between fitting and stability.(4) If actual observable data are not distributed evenly over a particular period of maturity, the polynomial would be prone to satisfactory goodness-of-fit in cases of shorter maturity of the curve and unsatisfactory goodness-of-fit in cases of its longer maturity, or *vice versa*. To address this problem, the polynomial degree  $k$  can be increased yet under a threat of reducing stability of the estimated parameters.

Economic intuition suggests that over a definite longer horizon expectations of market participants regarding the level of the nominal interest rates would converge towards a certain level. This suggestion is driven by the fact that beyond a longer time horizon market participants have less information to distinguish between the expected interest rate with maturity  $m$  and one with maturity  $(m + 1)$ . Hence in a longer maturity, instead of fluctuating the forward rate curves should become stable at a definite level.

In order to facilitate the trade-off between a good fitting quality and stability, the spline method can be used; according to it, the maturity spectrum is divided into a number of segments, and a separate relatively low degree polynomial is fitted for each such segment on the condition that at points where polynomials join (known as knot points) the estimated curve should be smooth: in the context of  $j$  order spline (the spline consists of  $j$  basis functions) it indicates that there is a derivative around each knot point. The spline is a curve that consists of some relatively low order polynomials. It produces a satisfactory fitting quality under retained stability.

The exponential spline, a modification of splines, reflects the discount function form as exponential. The following equation can be written about each pair  $d_m$  of the knot point:

$$d_m = e^{-rm - a_1m^2 - a_2m^3 - a_3m^4} \quad [2.5].$$

The spot rate curve estimation with splines has certain problems. The most serious of them consists in the fact that the number and location of knot points are chosen arbitrarily or, in the best case, according to some empirical rules. The number of knot

points determines the flexibility of the spline. A small number of points will result in a bad fit, whereas an excess number of points will impair the smoothness of the curve as is the case with the simple polynomial.

### *Stochastic Factor Methods and General Equilibrium Methods*

The theory of the interest rate term structure employs two groups of stochastic models – the factor models and the general equilibrium models. The focus in this study will not be on the theoretical aspects of the models, which are quite technical and complicated, but rather on the basic principles of the models.

The core purpose of the stochastic factor models is to explain the interest rate dynamics of securities with different maturities. To build the needed models, one or several random factors that explain the dynamics of short-term interest rates are selected as a starting point. The accepted stochastic process is specified to comply with the actual term structure. Specifying equilibrium conditions is the next step, providing a possibility of risk-free arbitrage and determination of a premium in accord with the maturity depending on a random factor. Using the factor model, partial derivations with an analytical solution of differentials for the bond price are formed.

General equilibrium models, in turn, are very close to the single-factor stochastic models. Factor models test price dynamics of financial assets separately (in a detached way) from overall economic indicators, and the stochastic process is freely chosen; the general equilibrium models, by contrast, deal with the general equilibrium condition of the whole economy. The stochastic nature of the financial asset price changes is determined by uncertainty of the basic economic indicators in the future. Consequently, the financial asset prices and their stochastic parameters are endogenous. Differential equation of the bond price is a component of the general equilibrium model.(35)

### *The Vasicek Single Factor Stochastic Model*

According to the Vasicek model, any information needed to describe a yield curve can be obtained using a single factor –  $r$  short-term spot rate.(33)

In O. Vasicek's opinion, the  $r$  short-term spot rate is constantly changing over time pursuant to the rules of normal distribution. The curve can be considerably low in some periods or considerably high in other, yet on the whole it has a tendency to return always to the average long-term value  $\varphi$ :

$$dr = \alpha(\varphi - r)dm + \sigma dz \qquad \alpha > 0 \qquad [2.6],$$

where  $r$  is the short-term spot rate;  
 $\varphi$  is the long-term average value of the rate  $r$ ;  
 $m$  is the time (in years);



$\sigma$  is the standard deviation (fluctuation factor);

$z$  is the Wiener process;

$\alpha$  is a constant parameter that represents magnitude with which the process returns to the long-term average  $\varphi$  and is proportional to the standard deviation of the valid spot rate value from the average ( $\varphi - r$ ).

Using the Vasicek model, the spot rate  ${}_0R_m$  is estimated by the following formula:

$${}_0R_m = \left( \frac{-A}{m} \right) + \left( \frac{B}{m} \right) r \quad [2.7],$$

$$\text{where } A = (B - m) R_\infty - \frac{\sigma^2 B^2}{4\alpha^2};$$

$$B = \frac{1 - e^{-\alpha m}}{\alpha},$$

$$\text{where, in turn } R_\infty = \varphi - \frac{\sigma^2}{2\alpha^2} + \frac{\lambda\sigma}{\alpha},$$

where  $\lambda$  is the market risk price.

The spot rate  ${}_0R_m$  is a linear function of the factor  $r$ . There are several adequately flexible models of this type cover different properties of the yield curve and give their description in time.

### ***The Cox–Ingersoll–Ross Model***

In 1985, J. Cox, J. Ingersoll and S. Ross found a solution for financial asset pricing within the model of general economic equilibrium and an alternative solution for determination of the interest rate term structure.(9)

Dynamics of interest rates is given by a stochastic differential:

$$dr = \alpha(\varphi - r)dt + \sigma\sqrt{r}dz \quad [2.8],$$

where notations correspond to those in the Vasicek's model.

Though the models of Cox–Ingersoll–Ross and Vasicek are based on different conditionality of equilibrium, their only principal difference is the inclusion of the square root from the variable  $r$  in the volatility parameter of the Cox–Ingersoll–Ross model with the aim to prevent the emergence of negative interest rates.

This model presents the theoretical spot rate  ${}_0R_m$  by the following formula:

$${}_0R_m = \left(\frac{-A}{m}\right) + \left(\frac{B}{m}\right)r \quad [2.9],$$

$$\text{where } B = \frac{2(e^{wm} - 1)}{D};$$

$$A = \left(\frac{2\alpha\varphi}{\sigma^2}\right) \left(\frac{(\alpha + w)m}{2} + \ln\left(\frac{2w}{D}\right)\right)$$

$$\text{and where, in turn } w = -\sqrt{\alpha^2 + 2\sigma^2};$$

$$D = (\alpha + w)(e^{wm} - 1) + 2w.$$

### ***The Nelson–Siegel Model***

S. Nelson and A. Siegel adopted the functional form of the forward rate curve.(28) They derived the functional form on the basis of an assumption that short-term forward rates are expressed by the following equation:

$$f_m = \beta_0 + \beta_1 e^{\frac{-m}{\tau}} + \beta_2 \left(\frac{m}{\tau}\right) \left(e^{\frac{-m}{\tau}}\right) \quad [2.10],$$

where  $f_m$  is the short-term forward rate for the period  $m$ , but  $\beta_0, \beta_1, \beta_2, \tau$  are parameters influencing the form of the forward yield curve.

Integrating the equation [2.10] from 0 to  $m$  and dividing the result obtained by  $m$ , a continuous spot rate for the term  $m$  is obtained:

$${}_0R_m = \beta_0 + (\beta_1 + \beta_2) \frac{\left(1 - e^{\frac{-m}{\tau}}\right)}{m/\tau} - \beta_2 e^{\frac{-m}{\tau}} \quad [2.11].$$

S. Nelson and A. Siegel adopted such a functional form for the yield curve, which is adequately flexible to present the shapes typical for the curve (increasing, inverted or hump-shaped) and used four parameters in its description. This specification offers interpretation of parameters as an advantage.

$\beta_0$  is a long-term (asymptotic) interest rate towards which the spot rate is converging. The sum of  $\beta_0$  and  $\beta_1$  is the instantaneous spot rate. The parameter  $\tau$  has an effect on velocity at which the spot rate will converge towards the asymptotic rate, whereas  $\beta_2$  is "humped" (or takes the U-form, if  $\beta_2$  is negative). The functional form selected by S. Nelson and A. Siegel satisfies requirements based on economic intuition about the

shape of the yield curve. This model became popular among the central banks because they are more interested in the basic shape of the yield curve than in a very accurate representation of the data. By contrast, other market participants often seek to obtain a yield curve that would represent market prices as accurately as possible to expose mispriced securities.

### *The Svensson Model*

To increase flexibility and fitting of the Nelson–Siegel yield curve, L. Svensson expanded the latter by adding an extra fourth parameter, another hump-shaped (or U-shaped) form  $\beta_3 \left( \frac{m}{\tau_2} \right) \exp \left( -\frac{m}{\tau_2} \right)$  with two additional parameters  $\beta_3$  and  $\tau_2$  ( $\tau_2$  must be positive) (31), and obtained the following function of the forward rate curve:

$$f_m = \beta_0 + \beta_1 e^{\frac{-m}{\tau_1}} + \beta_2 \left( \frac{m}{\tau_1} \right) \left( e^{\frac{-m}{\tau_1}} \right) + \beta_3 \left( \frac{m}{\tau_2} \right) \left( e^{\frac{-m}{\tau_2}} \right) \quad [2.12],$$

where  $\beta_0, \beta_1, \beta_2, \tau_1, \beta_3, \tau_2$  are parameters that affect the shape of the forward rate curve.

Integrating the equation [2.12] from 0 to  $m$  and dividing the result by  $m$ , gives a continuous spot rate for the term  $m$ :

$${}_0R_m = \beta_0 + (\beta_1 + \beta_2) \frac{\left( 1 - e^{\frac{-m}{\tau_1}} \right)}{m/\tau_1} - \beta_2 e^{\frac{-m}{\tau_1}} + \beta_3 \left( \frac{1 - e^{\frac{-m}{\tau_2}}}{m/\tau_2} - e^{\frac{-m}{\tau_2}} \right) \quad [2.13].$$

## **2.2 Yield Curve Modeling Experience of Central Banks of Other Countries**

The US and the UK developed their own traditions of the interest rate structure estimation long ago; in the 1990s, a number of the EMU central banks also started to evolve their own interest rate models. By 1997, in the construction of yield curves for the needs of their monetary policy central banks had broadly relied on the factor stochastic models, and the Nelson–Siegel model was the most popular among them.

The research done at the University of Vienna and the *Österreichische Nationalbank* using data of 1993–1998 (22) indicates that the estimation results obtained by the Nelson–Siegel method in Austria, Germany, the UK, the US and Japan on the whole are sound. For some periods of time, however, goodness-of-fit was not quite satisfactory. Due to it, the number of parameters in the Nelson–Siegel model was increased in compliance with the modeling methodology offered by L. Svensson in 1994. Experimenting with the Svensson model showed that adaptability could be improved

when an increased number of parameters are employed. Nevertheless, goodness-of-fit offered by L. Svensson was almost identical to the one obtained by the Nelson–Siegel model.

Since 1997, the Svensson model has been gaining in popularity, and a growing number of banks are using it in their pursuit of monetary policy basing on the following assumptions:

- 1) a uniform approach to the estimation of the term structure of interest rates in all countries encourages a more convenient comparison of them;
- 2) the Svensson model provides for a comparatively simple and sound means of estimation producing convincing yield curves, a feature significant for the monetary policy purposes. Employing the yield curve for such needs is not related to an extremely accurate pricing of financial instruments; instead, it is based on sustainability of the curve over a longer horizon. The Svensson model ensures it, and, in addition, offers a higher degree of data fitting than necessary in the analysis of monetary policy.

For instance, the ECB uses the Svensson model in the construction of the euro area's spot and forward rate curves.<sup>(15)</sup> It has also been used by the *Sveriges Riksbank* since 1994 and by the Bank of Canada since 1999. Prior to 1999, the Bank of Canada used a locally developed model in the construction of the yield curve, which was based on correlation of the nominal rate (*par yield*) and the yield to maturity using the LSM.<sup>(8)</sup> The *Deutsche Bundesbank* used the polynomial function of the yield curve between 1981 and 1997, which was constructed by means of the LSM through the existing yield to maturity data (without calculating the spot rate curve). From the point of view of calculating, this approach is simple, yet, as has been noted before, it has serious weaknesses if viewed from theoretical positions.<sup>(31)</sup> Like in a number of other countries, the Svensson model is used in the UK and the US, but alongside with it the Waggoner model using polynomial and spline methods is also employed.<sup>(5)</sup> Information available in developing countries is scarce, most likely because their government securities and money markets are new, liquidity is low and maturity terms of government securities are relatively short (often not exceeding 1 or 5 years). Due to it, the implied forward rates are difficult to estimate and their informative content is not reliable. The research of this kind in developing countries is just beginning. For example, the *Magyar Nemzeti Bank* was testing adaptability to the local securities market of the polynomial and spline methods as well as the Nelson–Siegel method and was recommended to employ the Svensson model in the construction of the government securities yield curve.<sup>(10)</sup>

### **2.3 Application of the Interest Rate Term Structure Modeling Methods to Processing of Latvia's Data**

In the construction of the spot rate curve for Latvia, this paper uses prices of the Latvian money market and the secondary market of government securities. These two market segments differ on various aspects. First, they differ on the risk factor

because interbank loans, predominantly unsecured, are the most widespread instrument on the money market, whereas government securities whose risk factor is close to zero are traded on the secondary market of government securities. This explains the eventual difference between the rates on interbank loans and government securities of the same maturity, as interest rates on interbank loans will also include additional credit risk evaluation. Second, the market of interbank loans is dominated by short-term instruments, basically loans issued overnight or with 7-day, 1- and 3-month maturity. The secondary market of government securities, in turn, is dominated by bonds with the term to maturity of 3 to 5 years. Taking account of these factors, the estimation of the spot and forward rate curves has been performed separately for Latvia's money market and the government securities market.

Of late, the Latvian money market has been developing dynamically. Loans issued in lats on the interbank market increased more than twofold in 1999–2002 (from 1.7 billion lats in 1999 to 3.8 billion lats in 2002). Interbank deals are of particular significance for the monetary policy pursued by the Bank of Latvia, as they enable the central bank, monopoly of the money supply, to control banks' crediting. This paper makes use of the money market interest rate index RIGIBOR as the source of information about interest rates of the Latvian money market. Its daily time series are accessible starting with December 8, 1997 when calculating of this index started. Taking account of conducting interbank market operations mainly at shorter terms of the yield curve (up to 1 month), it is difficult to analyse the actual market pricing of loans issued for 3, 6 and 12 months. Interest rates on loans in lats derived from the forward exchange rate quotations could be used as an alternative. It should be noted, however, that the rates obtained in this manner do not have a uniform index; in addition, prices of foreign currency resources may vary from bank to bank. That is why the use of other foreign currency rate indices, LIBOR as an example, will lead to inaccurate interest rates calculated on loans issued in lats. All above considerations lead to the choice of RIGIBOR for calculation purposes, because it is the most representative index on the domestic market of lats. Its dynamics is reflected by the data of Chart 2.1.

The evolution of RIGIBOR has not been smooth. The largest fluctuations were registered at the end of 1998 and 1999. Short-term interest rates (up to 1 month) have always showed pronouncedly larger fluctuations than long-term rates. This is quite reasonable, as it is the short-term transactions on the interbank market that are used in adjusting daily shifts in bank liquidity. Taking account of RIGIBOR dynamics, this study distinguishes three periods for which spot and forward rate curves are constructed (see Chart 2.1). The choice of the periods was determined by the interest rate fluctuations, which are losing their magnitude with each coming period.

The fluctuation of interest rates was most pronounced in the first period (December 8, 1997–March 31, 2000) due to a less liquid and effective money market of lats between 1997 and 1999. Moreover, the yield curve of this period has two notable bends. The first one at the end of 1998 was associated with the effects of the Russian finan-

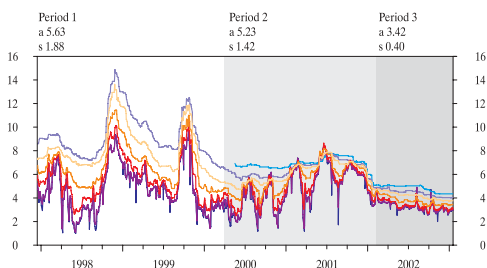
cial crisis. The domestic interbank market was subject to a cardinal change in 1998. The decision of the Government to deposit the major part of the budget funds with the Bank of Latvia brought about money resources reduction in the banking sector, which promoted activity of banks. This decision was a cause for a sizable decrease in the banking sector's excess reserves. From August 1998, the interbank market activity was affected by the 1998 financial crisis in Russia, which aroused mutual distrust among market agents. In 1998, the number of market participants shrank from 29 to 18, while interest rates were growing at the same time. More often banks ensured liquidity through borrowing funds from the Bank of Latvia. The second bend in the yield curve corresponds to the end of 1999. It was associated with the rapid growth in the government consolidated budget deficit and a swift rise in the banking sector's liquidity requirement, which emerged at the end of 1999 due to computer-related problems expected in early 2000.

Chart 2.1

**RIGIBOR DYNAMICS AND THREE PERIODS OF RIGIBOR TIME SERIES**

(08.12.1997–31.01.2003; figures are RIGIBOR 1-week average (a) and standard deviation (s); %)

- Overnight
- Tomorrow-next
- Spot-week
- 1-month
- 3-month
- 6-month
- 1-year



In the second period (April 1, 2000–January 31, 2002), less pronounced interest rate fluctuations than in the first period were registered. With macroeconomic factors remaining stable, fluctuations were mainly determined by foreign currency flows into the banking sector, currency interventions conducted by the Bank of Latvia and monthly seasonal factors (cash flows and flows of the government consolidated budget). The increased bank demand for lats was fostered by evolving lending activities and the expanding interest rate spread on the money market for transactions in lats and the OECD currencies, which was characteristic for the end of 2001.

In the third period (February 1, 2002–January 31, 2003), the money market interest rates were the most stable. The macroeconomic environment was sustainable, and the money market deepened and became more effective. In addition, liquidity of the banking sector was high in this period.

As has already been noted, spot rate curves for the Latvian securities market are also constructed. Government debt instruments play a notably larger part on the Latvian debt securities market than do the private debt instruments. The government securities outstanding were worth 293 million lats, and government eurobonds totaled 425 million lats at the end of 2002. Maturity of government debt instruments gradually

increased, and bonds with longer maturities were issued. The government started to issue bonds with 5-year maturity in 2000 and with 10-year maturity in February 2003.

Development of the government securities market in Latvia started in December 1993. Initially, almost all trading was conducted on over-the-counter market, but later – since 1999 – trading moved to the Riga Stock Exchange. The secondary market prices of the Latvian fixed income securities are available from the RSE and the electronic information system *Reuters*. Close to ten government debt instrument issues, forming the core of all government securities denominated in lats in circulation, are quoted there on a daily basis. With relatively low liquidity of the Latvian government securities in view, simultaneous transactions with all given securities are unlikely to happen. Hence the market prices used in the estimation are not the actual prices of transactions but the quoted buying prices. It may be argued whether the quoted buying prices give a real picture of the market prices, as every day a lesser number of securities are traded than quoted by the market participants. Moreover, traditionally market participants when buying and selling securities engage in direct deals, reaching a mutual agreement on the price and using the RSE infrastructure for settlements. Statistical data on actual prices of deals are not collected. That is why the upcoming estimations use quoted buying prices, which are available on a real-time, daily basis and whose daily time series can be obtained for the period of time since August 30, 1999.

Interest rates of securities with maturities from 2 months to 5 years, but since February 17, 2003 also up to 10 years, are mainly used in calculations. The government securities market has been dominated by the government bonds since the beginning of 2000. Their proportion in the securities market structure has increased from 20% in April of 1997 (when government bonds with 2-year maturity were first issued) to 90% at the end of 2002. As has been noted before, bonds with the initial maturity of 3, 5 and 10 years are traded currently. At the end of 2002, around 40% of the traded government securities were those with up to 1-year redemption term. As bond issuances are not organised on a monthly basis and auctions are not held regularly, the quotation data are not spread evenly on the yield curve. For instance, at the end of 2002, there were several breaks in the quotation of securities with time to maturity from 10 months to 2 years, and from 3 to 4 years. With the issuance of 10-year bonds, a quotation break has formed also for securities with time to maturity from 5 to 10 years.

Similar to money market calculations, the following time series periodicities have been used in testing the spot and forward rate curves obtained: the first period from August 30, 1999 to March 31, 2000; the second period from April 1, 2000 to January 31, 2002 and the third period from February 1, 2002 to January 31, 2003.

On the basis of Latvia's data, discount, spot and forward rate curves are constructed using alternative approaches, i.e. the third degree polynomial calculation, the exponential spline model and Nelson–Siegel, Vasicek, Cox–Ingersoll–Ross and Svensson models.

Which is the best model? There are a number of criteria to be observed in the decision-making about the approach to be used in discussing the term structure of interest rates of any financial market. The most important of them are:

- 1) fitting. The spot rate curve is to reflect observable data with adequate accuracy. It shall quickly respond to changes in the situation through adjusting to the real shifts in the term structure;
- 2) stability. The spot rate curve must be insusceptible to changes in yields on individual bonds. Shifts in the market data of one or several bonds shall not affect the yield on other bonds;
- 3) practical application. The shape of spot and forward rate curves shall be consistent with particular economic intuition. Sloping of the spot rate curve should be homogeneous, whereas the discount curve must not have a positive slope;
- 4) consistency with criteria over longer horizon. The spot rate curve shall meet the criteria in a longer period of time, not just one particular day;
- 5) comparability of a model with those of other countries. The more countries use the same models, the easier it is to compare estimation outcomes in different countries.

Theoretically estimated price (or yield) is obtained by minimising deviations between theoretical and actual bond prices (or yields). Three basic indicators can be used:

- 1) Root Mean Squared Error, RMSE;
- 2) the determination coefficient  $R^2$ ;
- 3) the determination coefficient  $R^2$ , adjusted for degrees of freedom (i.e. the adjusted  $R^2$ ).

RMSE is a minimisation criterion that is used in estimations; it figures as the core comparability criterion for various methods of modeling. The number of included parameters is the determining factor for elasticity of methods used in curve modeling. With other factors unchanged, models with a larger number of parameters adjust to observable data better. For example, a model with the same number of parameters and observables adjusts theoretical data to observable data perfectly.

Fitting is the level up to which an estimated curve can reflect fluctuations of observable prices. A small RMSE and a large coefficient  $R^2$  and the adjusted  $R^2$  are indicators of high fit. An excessive fluctuation observed in a calculation is not to the advantage of practical application of the estimated curve and, consequently, also its ability to capture real changes in market expectations. In any case, a balance between stability and fitting should be sought. Where models with a larger number of parameters are used, data can be described more accurately and a lesser gap between the estimated (theoretical) and the observable yield is obtained. These methods can prove important for those market participants who engage in securities trading in which the interest rate structure is used as a basis for evaluating securities. On the other hand, monetary policy specialists of central banks usually give preference to a more stable curve, e.g. in cases when the interest rate term structure is used as an indicator of money market expectations implying future interest rate or inflation tendencies. The



less important the fitting, the greater simplicity of estimation methods and outcome interpretation can be obtained. However, an exaggerated trade-off between fitting and simplicity is not desirable, because the curve can be pressed so much that it loses its market expectations-related informative content.

### Results of Empirical Estimation of the Money Market

Empirical results of three observable periods used in the estimation of the money market are given in Appendix 1.

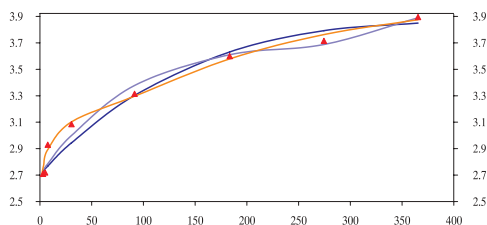
Charts 2.2 and 2.3 reflect spot and forward rate curves of the Latvian money market obtained by various methods of modeling as at March 25, 2003.

Chart 2.2

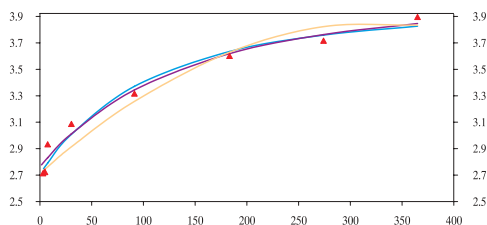
#### SPOT RATE CURVES FOR THE LATVIAN MONEY MARKET

(obtained 25.03.2003 by various modeling methods; RMSE for each model is given in parentheses; in days; %)

- Exponential spline model (0.07)
- Vasicek model (0.07)
- Svensson model (0.03)
- ▲ RIGIBOR



- Nelson–Siegel model (0.06)
- Cox–Ingersoll–Ross model (0.07)
- Polynomial model (0.10)
- ▲ RIGIBOR



The assessment of empirical results obtained indicates that all the models produce smaller RMSEs with every coming reporting period (see Appendix 1). All assessed methods of modeling display a comparatively good coefficient  $R^2$  and the adjusted  $R^2$  coefficient. Such excessively good results are likely to be associated with a relatively small number of 8 financial instruments.

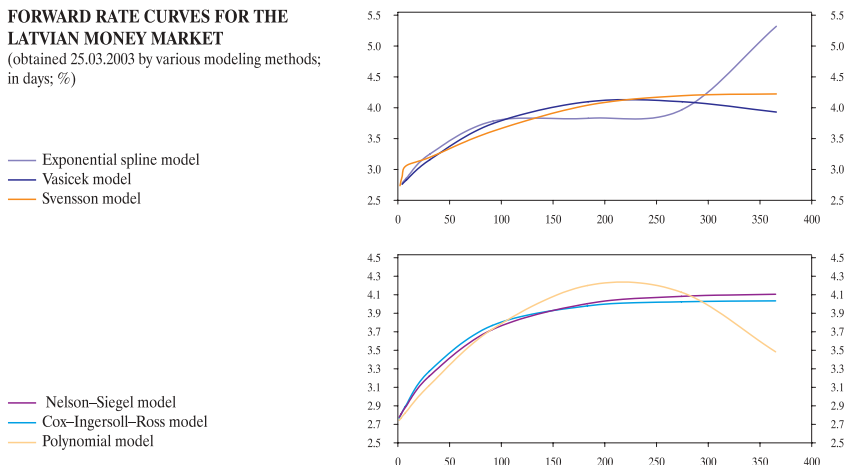
Testing results indicate that the Svensson model is the most favorable one for all three periods. On the one hand, it offers the lowest value of RMSE and the largest coefficient  $R^2$  and the adjusted  $R^2$ ; in all three reporting periods, this model is, likewise, referred to as the best from the point of view of RMSE, i.e. it meets the goodness-of-fit criterion in a longer period of time. The results initially produce a positive impression and confirm that the Svensson model, which has been gaining popularity with many central banks since 1996, suits the Latvian money market well. Success of the

Svensson model on the Latvian money market is determined by a larger number of parameters in comparison with other models. More parameters require more time for calculation, yet this is not a serious weakness. On the other hand, testing disclosed some weaknesses with regard to the Svensson model's applicability to the Latvian government securities market. These drawbacks do not permit the Svensson model to be acknowledged as the most suitable one for simultaneous analysis of both the Latvian money market and the government securities market.

Chart 2.3

**FORWARD RATE CURVES FOR THE LATVIAN MONEY MARKET**

(obtained 25.03.2003 by various modeling methods; in days; %)



A second best model from the point of view of RMSE and  $R^2$  is the exponential spline model. Similarly, it also has some qualities that do not permit to accept it as the best for the yield curve modeling using Latvia's data. Estimations show that the spline and polynomial models can prove problematic because they do not produce a feasible and stable form of spot and forward yield curves, and, consequently, fail to meet the criterion of practical applicability. For example, polynomials and splines of higher degree have certain periods for which the discount function becomes negative. For the days of such periods, therefore, spot rate curves cannot be calculated using the given methods. The problem of a negative discount function can be avoided if the factor and the general equilibrium stochastic models are used, hence economic significance of the parameters of stochastic models can be an advantage. The factor stochastic methods guarantee better practical applicability and sustainability of both the spot and the forward rate curves as they correspond to a definite degree of economic intuition over a longer horizon. The three stochastic methods described hereinbefore differ insignificantly: the Nelson-Siegel, the Cox-Ingersoll-Ross and the Vasicek models rank after the exponential spline with about the same results, yet the Nelson-Siegel model displays slightly better results in all three periods, hence it may be selected as the most suitable for the current Latvian money market.

## Results of Empirical Estimation of the Securities Market

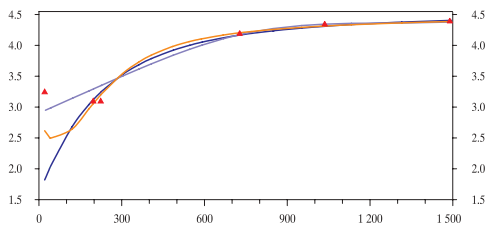
Appendix 2 presents the spot rate curve summary statistics of the Latvian government securities market. The curves have been derived by various methods of modeling (see Charts 2.4 and 2.5).

In contrast to the money market results, those of the securities market are not so homogeneous. In the initial development periods of the securities secondary market, the best results were obtained using Svensson, Vasicek and Cox–Ingersoll–Ross models.

Chart 2.4

**SPOT RATE CURVES FOR THE LATVIAN GOVERNMENT SECURITIES MARKET**  
(obtained 25.03.2003 by various modeling methods; RMSE for each model is given in parentheses; in days; %)

— Exponential spline model (0.24)  
— Vasicek model (0.68)  
— Svensson model (0.23)  
▲ Quotation rate



— Nelson–Siegel model (0.47)  
— Cox–Ingersoll–Ross model (0.84)  
— Polynomial model (0.24)  
▲ Quotation rate

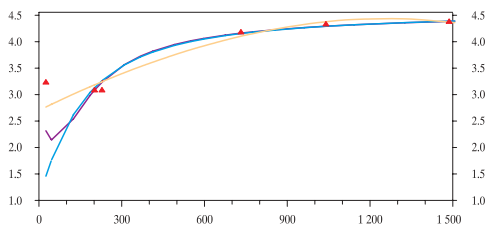
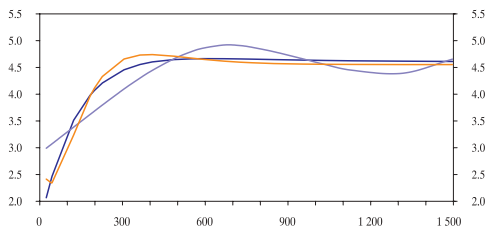


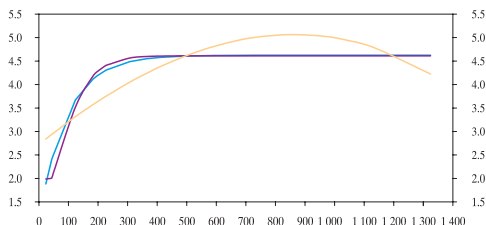
Chart 2.5

**FORWARD RATE CURVES FOR THE LATVIAN GOVERNMENT SECURITIES MARKET**  
(obtained 25.03.2003 by various modeling methods; in days; %)

— Exponential spline model  
— Vasicek model  
— Svensson model



— Nelson–Siegel model  
— Cox–Ingersoll–Ross model  
— Polynomial model



With the market stabilising, the application of polynomials and splines also produces ever-improving results. It should be noted, however, that similar to the money market these methods have a drawback consisting in very unstable results for certain days. For example, the exponential spline adjusts the yield curve to the actual data fairly well on a number of occasions, yet there are cases where it becomes particularly responsive to changes in the yield of particular bonds producing large deviations and hence also RMSE.

As has already been noted, such a problem, as a rule, does not emerge when factor stochastic methods are used. Estimations by Svensson, Nelson–Siegel, Vasicek and Cox–Ingersoll–Ross models indicate that the long-term interest rates in March 2003 were predisposed to move towards the average equilibrium of 4.6%–4.8%.

Similar to the money market, the Svensson model is ranked as the best for all three securities market periods from the point of view of RMSE. However, this model is more sensitive to differentiating results of securities quotation than other stochastic models and, therefore, often produces a biased term structure or even proves to be unable to arrive at one, i.e. does not comply with the stability criterion. With relatively low liquidity of the secondary market of government securities in view, some interest rates quoted are far from the constructed spot rate curve, which, in turn, puts restraints on the application of the Svensson model. As the aim is to find a single model for the estimation of both the money market and the government securities market, the Svensson model currently is not the best alternative mainly due to inadequate development of the secondary market of government securities.

Though a slightly larger RMSE is typical for other stochastic models, they operate quite successfully if the entire set of estimation criteria, including compliance in a longer period, is considered. The difference is relatively small. At this point, it is difficult to state with assurance which method reflects the Latvian government securities market data best. If judged from the position of the curve stability, preference could quite likely be given to the Cox–Ingersoll–Ross or the Nelson–Siegel approaches, yet one should always bear in mind risks associated with elasticity of multiparameter models. Problems may arise when on the market of government securities spot rates are calculated for short-term government securities that are inadequately or not at all reflected by the actual information.

The outcomes of this paper indicate that for the needs of the Bank of Latvia, the Nelson–Siegel model is a more suitable approach in the estimation of spot rate curves of the Latvian money and government securities markets. Its preference is also determined by the widespread use of its extended form (the Svensson model) by central banks of developed countries. Furthermore, a number of studies indicate that the Nelson–Siegel model is functioning on developed markets as successfully as is the Svensson model, with a shorter time needed for estimations and lower sensitivity to outliers as its advantages.

Currently at the Bank of Latvia, yield curves are constructed for RIGIBID/RIGIBOR, Latvian government securities, LIBOR EUR, LIBOR USD, LIBOR GBP and synthetic LIBOR SDR interest rates. The parameters of each model derived by interpolation are stored in the database and are accessible for further estimations.

### 3 THE INTEREST RATE EXPECTATIONS THEORY

#### 3.1 Essence of the Interest Rate Expectations Theory and Basic Principles of Its Testing

As has been noted in Chapter 1, under certain circumstances the yield curve captures information about interest rates expected in the future. The objective of numerous empirical studies is to test possibilities of applying the interest rate expectations theory. Provided the testing of the theory is supported by financial data, the yield curve can be used for monetary policy purposes as an instrument for studying expectations of market participants with regard to interest rates in the future.

For example, short-term forward rates can be interpreted as implying the market expectations in respect of short-term interest rates at various future periods; it should be borne in mind, however, that forward interest rates will be the same as the expected short-term future interest rates only under the condition that no risk premiums exist, i.e. if the pure interest rate expectations theory holds. If it is rejected, the implied forward rates contain a risk premium, which is likely to present bigger problems for interest rate forecasting. However, if the risk premium for a definite term is constant, the interest rate expectations theory with a constant risk premium holds, and this enables forecasting of respective interest rates. In case the expectations theory with a constant premium is rejected, the information content of the yield curve is not of great significance.

The question whether the interest rate expectations theory holds is an empirical one and can be tested by econometric methods. The quantitative formulation of the pure interest rate expectations theory is as follows (symbols will slightly differ from those in Chapters 1 and 2):

$${}_{t+m}r_j = {}_t f_{j,t+m} \tag{3.1}$$

where  ${}_{t+m}r_j$  is the interest rate in period  $(t+m)$  for  $j$  maturity;

${}_t f_{j,t+m}$  is the forward rate in period  $t$  for period  $(t+m)$  for  $j$  maturity;

$m$  is the forecasting horizon.

For the purpose of testing equation [3.1], the following regression shall be made using historical data:

$${}_{t+m}r_j = \beta_j \times {}_t f_{j,t+m} + \alpha_j \quad [3.2],$$

where  $\alpha_j$  is the risk premium.

In compliance with the pure interest rate expectations theory, the current forward rate is, on average, similar to the future spot rate. Hence it is assumed that the coefficient  $\beta_j = 1$ , while  $\alpha_j = 0$ . In order for the yield curve to have forecasting ability (even a weak one), the coefficient  $\beta_j$  should be statistically significant (i.e. significantly different from zero).

The following hypotheses are proposed for testing the interest rate expectations theory.

1.  $H_0: \beta_j = 0$  is the hypothesis about forecasting ability of the yield curve. If  $\beta_j \neq 0$ , the term structure of interest rates contains information about future interest rates, and equation [3.2] may be useful for interest rate forecasting. If the hypothesis  $H_0: \beta_j = 0$  is rejected, a next one is tested.

2.  $H_0: \beta_j = 1$  is the hypothesis about the existence of a constant premium. If  $\beta_j = 1$  is statistically significant, the hypothesis about a constant premium is accepted, which implies that the interest rate expectations theory holds. In case this hypothesis cannot be rejected, a more complex hypothesis is tested.

3.  $H_0: \beta_j = 1, \alpha_j = 0$  is the hypothesis about the existence of a zero premium. If  $\beta_j = 1, \alpha_j = 0$  are statistically significant, the pure interest rate expectations theory holds.

Summary of the hypotheses proposed for testing the expectations theory is given in Appendix 3.

Usually, the interest rate expectations theory is tested by an equation that slightly differs from equation [3.2]. In developed countries, the time series of interest rates are non-stationary, as statistical characteristics of interest rate fluctuations change with time. All variables should be stationary – this is a precondition of the regression theory. All up-coming conclusions on coefficients  $\alpha_j, \beta_j$  as unbiased, convergent and effective are based on the assumption about stationary variables. In order to obtain stationary processes, differentiated data are usually used. In most cases, these differences are stationary. Hence the expectation theory is tested as follows:

$${}_{t+m}r_j - {}_t r_j = \beta_j ({}_t f_{j,t+m} - {}_t r_j) + \alpha_j + {}_{t+m} \varepsilon_j \quad [3.3],$$

where  ${}_{t+m} \varepsilon_j$  is the regression forecasting error due, for instance, to an economic shock from period  $t$  to period  $(t + m)$ .

Another problem arising in the process of assessing the forecast regression is that of potential serial correlation of residuals. Residuals will be subject to serial correlation at  $m > 1$  forecast depth. The error of the forecast occurs only in period  $(t + m)$ . Hence  ${}_{t+m} \varepsilon_j$  will be correlated with the errors of  ${}_{t+1} \varepsilon_j, {}_{t+2} \varepsilon_j, \dots, {}_{t+m-1} \varepsilon_j$ . Thus the estimations

by ordinary LSM will not be converged. For the purpose of this study, the coefficients of the forecast regression are estimated by the Newey–West methodology.

If it is assumed that the hypothesis  $\beta_j = 1$  is not rejected for regression [3.3], on the basis of equation [3.3] we arrive at:

$${}_{t+m}r_j - {}_t r_j = {}_t f_{j,t+m} - {}_t r_j + \alpha_j + {}_{t+m} \varepsilon_j \quad [3.4]$$

$$\text{or } {}_{t+m}r_j = {}_t f_{j,t+m} + \alpha_j + {}_{t+m} \varepsilon_j \quad [3.4a].$$

Thus the future interest rate  ${}_{t+m}r_j$  is the forward rate  ${}_t f_{j,t+m}$ , with the constant premium  $\alpha_j$  added to it.

The interest rate expectations theory has been widely discussed in various research papers; the empirical evidence, however, differs from study to study depending on the hypothesis, yield curve segment and the time period tested. At this point, two surprising results deserve attention. First, the interest rate expectations theory is almost unanimously rejected in the US financial market studies. Second, market studies outside the US differ significantly, yet the majority of them do not reject this theory.

For instance, G. A. Hardouvelis, when examining 3-month and 10-month interest rates in G-7 countries, discovered that the interest rate expectations theory gives a comparatively poor explanation of the interest rates in the US, while those of other countries are comparatively well reflected.(23) Further confirmation regarding a more problematic rejection of the interest rate expectation theory for non-US financial markets has been provided by S. Gerlach and F. Smets (1997) when they studied interest rates of 1-, 3-, 6- and 12-month maturities in 17 countries in the period between 1979 and 1996.(20) Their studies demonstrated that the interest rate expectations theory is difficult to reject in the majority of cases, yet it can be rejected when dealing with the US interest rates of all maturities. Generalisation of the outcomes enabled S. Gerlach and F. Smets to conclude that better examples of the expectations theory in the European countries in comparison with the United States can be explained by the fact that the former countries operated in the fixed exchange rate regimes for the largest part of the reporting period and experienced periods of speculative pressure on the currency markets. An essential temporary increase of interest rates with a subsequent gradual return to the former level is a typical reaction of central banks to speculative pressures. S. Gerlach and F. Smets believe that the combination of such speculative pressures and consistent and regular reaction of central banks fosters predictability of short-term interest rates. This assumption is supported by a study conducted by E. Jondeau and R. Ricart on the basis of interest rates of the French franc, which shows that testing of the interest rate expectations theory is perceptive of the data of speculative pressure periods (1981 and 1983) included in the model.(25) S. Gerlach and F. Smets do not believe that speculative attacks figure as a sole reason for the expectations theory being difficult to reject in countries with the exchange rate anchor. A. Hurn, T. Moody and V. Muscatelly obtained positive results for the expect-

tations theory from the United Kingdom interbank market interest rates using the same method.(24) M. Dahlquist and G. Jonsson, when dealing with the interest rates of the Swedish government securities, were unable to reject the interest rate expectations theory.(11)

In respect of interest rate term structure studies in developing countries, testing of the interest rate expectations theory conducted in Argentina (L. Leiderman and M. Blejer)(26), Mexico (I. Domowitz, J. Glen and A. Madhavan)(14) and Russia (S. Drobishevskij)(35) deserves particular attention. These studies indicate that properties of the interest rate term structure pertaining to financial markets of the developing countries on general lines correspond to the results of the developed countries, and the actual long-term interest rates contain information about the future short-term interest rates.

### **3.2 Testing of the Interest Rate Expectations Theory on the Latvian Financial Market**

As mentioned above, a necessary precondition for evaluating the regression is stationarity of the time series. Thus, before testing the interest rate expectations theory, stationarity of interest rate time series of the money and securities markets should be tested, i.e. the unit root test should be conducted.

For stationary testing, the Philips–Perron test for the interest rate time series and the first-order difference of the time series under review was carried out separately for the Latvian money market and the government securities market. In view of the fact that the Latvian money market and the securities market have differed considerably at various times since their establishment, three sub-periods were selected for testing the time series' stationarity. To a great extent, they coincide with those referred to in Chapter 2, yet there are also slight differences, which have been determined partly by the results of stationary tests and partly by the intention to compare interest rate forecasts obtained in 2002 and 2003 using the actual interest rates. The following periods are viewed in respect of the money market:

Period 1: December 8, 1997–March 31, 2000;

Period 2: April 1, 2000–January 31, 2002;

Period 3: February 1, 2002–November 29, 2002.

The following periods are viewed in respect of the government securities market:

Period 1: August 30, 1999–March 31, 2000;

Period 2: April 1, 2000–October 31, 2001;

Period 3: October 1, 2001–September 30, 2002 (a slight overlapping with Period 2 due to stationary results).

For all periods, tests were conducted on the basis of daily and weekly time series. All interest rate time series assessed have been obtained from the data time series using the Cox–Ingersoll–Ross model.



The choice of interest rate time series of particular maturity for stationarity testing depends on which maturities and forecasting horizons the expectations theory is tested. The maturity of interest rates and forecasting horizons selected for testing in this paper are shown in Charts 3.1 and 3.2.

Chart 3.1

**MATURITIES SELECTED FOR STATIONARITY TESTING AND FORECAST HORIZONS FOR THE LATVIAN MONEY MARKET**

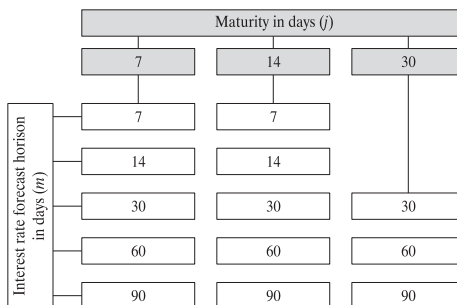
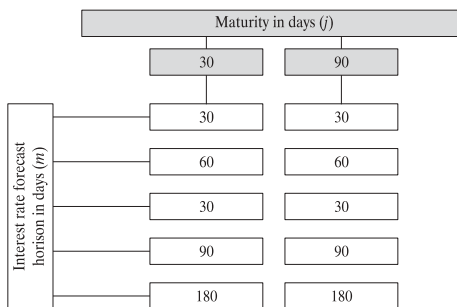


Chart 3.2

**MATURITIES SELECTED FOR STATIONARITY TESTING AND FORECAST HORIZONS FOR THE LATVIAN GOVERNMENT SECURITIES MARKET**



According to the Philips–Perron test, the interest rate time series are non-stationary in most cases, while their data differences are stationary.

After the estimation of stationarity, the interest rate expectations theory itself was tested. The results of stationary testing indicated that 58 regressions – 32 for the money market and 26 for the securities market – were accepted as suitable for the expectations theory testing. It should be noted that the maximum maturity horizon of the government securities is 6 months because interest rate time series are non-stationary for longer forecasting horizons.

**Results of the Money Market Review**

Appendix 4 presents estimations of the coefficient  $\beta$ ,  $t$ -statistics, regression determination coefficients and results of related hypotheses. For Period 1, the hypothesis of

the yield curve's forecasting ability can be rejected in all cases, as under a relatively large number of observations it cannot be rejected that  $\beta = 0$  (95% significance). In addition, the results obtained for the daily and weekly data time series do not differ noticeably. Similarly, the forecasting ability of the yield curve is weak for Period 2 as well. The hypothesis of the existence of a constant risk premium cannot be rejected only in one regression of the four given. Overall for Period 1 and 2, however, the majority of interest rate time series turned out to be non-stationary, hence the related hypotheses cannot be estimated.

In Period 3, the yield curve indicates a good forecasting power for different maturities over different forecasting horizons. The maximum horizon studied is up to 3 months. The hypothesis that  $\beta = 0$  can be rejected for all maturities and forecasting horizons, and it implies that the yield curve has some forecasting power in respect of future interest rates for up to a 3-month horizon. The hypothesis that  $\beta = 1$  is not rejected, whereas the one simultaneously restricting  $\beta$  and  $\alpha$  is, by contrast, rejected. It implies that the interest rate expectations theory in its pure form cannot be used for the Latvian money market; on the other hand, it can be applied in a form that does not require a perfect conformity of the forward and future spot rates.

In Period 3, estimations basing on weekly data time series generally produce better results than those building on daily data time series. It is evidenced by the  $\beta$  coefficient values, which are fluctuating around 1 (from 0.8 to 1.3). In addition, the hypothesis that  $\beta = 1$  cannot be rejected using  $p$ -value, which for different equations fluctuates within the range of 0.23–0.62 in the majority of cases. A relatively higher value of the determination coefficient  $R^2$  also testifies to considerably good forecasting properties.

It should be noted, however, that the value of the Durbin–Watson statistic is predominantly low at 0.3–0.4 for daily data and slightly higher for weekly data (0.9–2.2), yet mainly below 2, which is an indicator of a serial correlation in residuals and a factor reducing forecasting efficiency. The solution of the correlation problem in residuals, as noted before, is based on the Newey–West methodology. Yet, in the event of the forecasting horizon  $m$  exceeding the interval between observations, the problem of correlation in residuals still persists. The larger the difference between the forecasting horizon and the given time interval, the smaller is the value of the Durbin–Watson statistic. On the other hand, when the forecasting horizon is close to or coincides with the time interval between observations, the Durbin–Watson statistic has a better value (closer to 2). This explains why for interest rate forecasts for 7 and 14 days using weekly data series, the Durbin–Watson statistic value is larger than for the respective 30- and 60-day forecasts.

### ***Results of the Securities Market Review***

The results of the regression estimation are given in Appendix 5. For Period 1, the possibility to use the yield curve in forecasting can be rejected almost completely. For

Periods 2 and 3, better results, though rather volatile for different terms and forecasting horizons, are obtained. On the whole, the results are contradictory to the pure interest rate expectations theory, because forward interest rates do not ensure unbiased forecasts of spot interest rates in the majority of cases. The only result without the rejection of the hypothesis that  $\beta = 1$ ,  $\alpha = 0$  at  $p$ -values of 0.09 and 0.18, and, hence, also the related hypothesis about the zero risk premium was obtained for Period 2 when forecasting 1-month spot interest rate for one month forward and 3-month interest rates for three months forward using daily data time series. The given result, however, cannot be treated unequivocally. For the same period, when forecasting 1-month and 3-month interest rates for two months forward, negative coefficients  $\beta$  not consistent with the interest rate expectations theory were obtained; in both instances, however, the negative coefficients are statistically insignificant. They are likely to be associated with a large number of shock situations that impair the shape of the yield curve.

The results of Period 2 using weekly time series demonstrate that in all cases the hypothesis that  $\beta = 0$  can be rejected, hence the forecasts using weekly time series produce comparatively better results than the forecasts using daily time series. Such results can be attributed to the weekly time series having fewer accidental deviations. The results indicate that the hypothesis about a constant risk premium cannot be rejected, for instance, for a 3-month interest rate forecast for six months forward where  $\beta$  is close to 1 ( $\beta = 1.075$ ), while the hypothesis that  $\beta = 1$  cannot be rejected with high probability (at  $p$ -value of 0.8029).

The results for Period 3 also indicate that the hypothesis  $\beta = 0$  can be rejected for all maturities and forecasting horizons given. By contrast, the hypothesis about the existence of a constant risk premium cannot be rejected using daily time series for 1-month and 3-month interest rate forecasts for one month forward, and for 3-month interest rate forecast for two months forward, as well as for 1-month interest rate forecast for one month forward using weekly time series.

The results of testing the interest rate expectations theory obtained for different markets – money and securities – at different periods are quite distinguishable. Nevertheless, they lead to certain inferences characterising the interest rate term structure on the Latvian money and securities markets.

Overall, the results reject the pure interest rate expectations theory of the term structure, since the forward interest rates on the whole (with only few exceptions that cannot be considered general) do not ensure unbiased spot rate forecasts. This outcome is logical, as it indicates that forward interest rates contain a risk premium other than zero, i.e. long-term interest rates are higher in comparison with the expected future short-term average interest rate, since investors demand a risk premium that is positively dependent on the maturity. It is noteworthy that also the developed financial markets mostly reject the pure interest rate expectations theory.

In general, the assumption regarding rationality of the market participants is comparatively firm and in respect of the Latvian financial market can be attributed only to separate periods of time since 2000. In this respect, the situation on the Latvian money market is better because since April 2000 the hypothesis about the existence of a constant risk premium cannot be rejected in majority of cases when forecasting 7-, 14- and 30-day rates for the period of up to two months forward. The empirical results obtained for the Latvian securities market, on the other hand, are not so convincing, as in more than half of the cases, when analysing 1- and 3-month forward rates for the time period of up to six months forward, the hypothesis with a constant risk premium can be rejected with a very small error probability.

Such distinguishable money and government securities market results can be explained by the differing liquidity levels of the two, and also the quality of the interest rates quoted. Of late, the money market, which by its nature is an interbank market, has advanced significantly in terms of both the turnover and the number of instruments used, whereas in respect of the securities market, which has also grown in terms of the volume and maturities, the turnover of securities on the secondary market is still rather small. Liquidity on both money and government securities markets is well characterised by the so-called liquidity ratio, which is calculated as the average turnover of a certain financial instrument against the average stock of this instrument in a definite period (see Table 3.1). The liquidity of the Latvian government securities market is extremely low, while that of the developed financial markets in France, Sweden, the US and the UK is 33.8, 32.7, 22.0 un 7.0, respectively.(30)

Table 3.1

**LIQUIDITY OF THE LATVIAN GOVERNMENT SECURITIES AND INTERBANK CREDIT MARKETS**

	2000	2001	2002
<b>Government securities market:</b>			
Average stock (in millions of lats)	209.2	279.6	270.3
Turnover (in millions of lats)	351.1	406.2	177.2
Liquidity indicator	1.68	1.45	0.66
<b>Market of domestic interbank credits in lats:</b>			
Average stock of claims (in millions of lats)	18.6	46.2	63.6
Turnover (in millions of lats)	2 082.1	3 435.1	3 776.8
Liquidity indicator	111.9	74.4	59.4

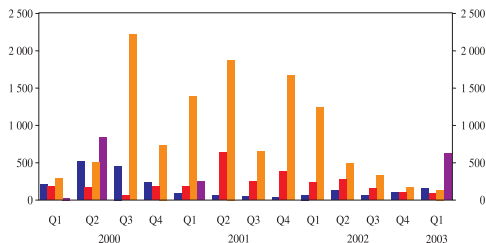
Obviously, at low market liquidity the market prices of government securities quoted cannot be taken as a quality source of information for the assessment of interest rate term structure. Furthermore, it is difficult to speak about rationality of expectations inherent in this term structure, if trading on the secondary market takes place only in the longer horizons of the yield curve (with maturity exceeding three years) but for shorter maturities it may cease for a longer period of time (see Chart 3.3).

Chart 3.3

**TURNOVER OF THE LATVIAN  
GOVERNMENT SECURITIES  
SECONDARY MARKET**

(by maturity to redemption; in thousands of lats)

- Up to 1 year
- 1–3 years
- 3–5 years
- Over 5 years



Nevertheless, empirical results provide for an adequate amount of statistically significant positive coefficients. Evaluations of the coefficient  $\beta$ , though below 1, are positive (95% significance). This allows for accepting the interest rate expectations theory in a weaker form that does not require a precise concurrence of the forward and future spot rates. Thus the hypothesis of the forecasting ability of the yield curve is not rejected. The contradictions above notwithstanding, the estimations obtained lead to an assumption that forward rate curves contain information on the future spot rates. However, the accuracy of such forecasts is low, and in most cases forward rates are biased ( $\alpha \neq 0$ ) future spot rate evaluations.

On the Latvian financial market, the yield curve forecasting ability often gives in to that estimated for the euro area. This is not surprising, as in addition to liquidity considerations referred to above the Latvian money and government securities markets are still immature. Moreover, the period studied was rich in various shock situations and most likely also a notable risk premium volatility, which collectively impaired the process of the yield curve forecasting.

A low  $R^2$  and also low Durbin–Watson statistical values indicate that the forecasting power is unsatisfactory, hence there are no grounds for making formal inferences regarding statistical significance of the results obtained. It should be emphasised that inferences regarding forecasting abilities of the yield curve essentially depend on the choice of the length of the time series period. This, in turn, is associated with the amount of the risk premium in investors' forecasts that may change over time. It is clear that in cases when the risk premium is not constant, the interest rate expectations of market participants cannot be considered rational. Therefore, risk premium variance in time is likely to explain why the forward interest rates are biased future spot rate estimations in the majority of cases.

#### **4 APPLICATION OF THE INTEREST RATE TERM STRUCTURE TO THE BANK OF LATVIA'S MONETARY POLICY**

This Chapter makes an attempt to link the empirical results obtained on the yield curve modeling and practical testing of the interest rate term structure on Latvia's

market, on the one hand, and the Bank of Latvia's monetary policy, on the other, as well as to outline other eventual areas of study using the yield curve of the Latvian financial instruments.

How can a central bank make use of the information contained in a yield curve for monetary policy purposes? First, the most straightforward and obvious objective is determination of prices for various financial instruments, as the central bank acts on the market and engages in various money and securities operations like any other participant. Using daily database, the Bank of Latvia is already assessing government and private fixed income securities, which it has accepted as collateral for its repo deals with banks, and its own fixed income securities portfolio. The Bank of Latvia evaluates securities using interest rates on respective issues of securities quoted at the RSE on the previous business day. As respective securities are not always quoted at the RSE, a spot rate curve interpolating a missing quotation and permitting price determination for respective securities is to be constructed. Currently, the Bank of Latvia employs the Nelson–Siegel model in the construction of the yield curve because it has been one of the most favored methods of central banks at different times. The comparative analysis of modeling methods performed for various yield curves in this study enabled us to conclude that the application of the Nelson–Siegel model to the Latvian government securities market is justified on the grounds that the reflection of actual data is sufficiently accurate, and stable and plausible spot rate curves for longer periods of time can be constructed. The empirical results obtained by this study indicate that the Svensson yield curve model (the expanded Nelson–Siegel model) employed by the ECB and the majority of central banks in the EU countries cannot, for the time being, be applied to the Latvian government securities market data. It is, however, noteworthy that despite the factor stochastic and general equilibrium yield curve models being considered an instrument guaranteeing a reliable result, it is not excluded that at some point yield curves with unsatisfactory properties and hence not employable in the assessment of government securities on certain days would be constructed.

The yield curve can also be used in the analysis of information contained in it for the purpose of timely identification of market participants' expectations and confidence in the current monetary policy. The significance of such analysis performed at the Bank of Latvia is likely to increase in the near future under the impact of recent advance of the Latvian money and government securities markets. It is confirmed by the rising level of market liquidity, increasing number of participants and expanding range of financial instruments on the market, bringing about also improvements in the quality of interest rates quoted. In the given circumstances, it would be appropriate for the local financial market to commence the testing of more complicated methods of analysis, which are already used on developed financial markets. The assessment of empirical properties of information contained in the prices of financial assets is one of the central trends of contemporary financial market research. The study of

the Latvian financial market indicates that since 2001 and 2002, the yield curve has had a certain forecasting power in compliance with the interest rate expectations theory, and it implies that forward rates calculated under certain assumptions can be interpreted as implying short-term interest rates expected in the future.

The growing role of analysis will also be determined by the approaching integration of Latvia with the EMU. Upon Latvia becoming a part of a single economic and monetary area, the significance of the interest rate channel in Latvia's monetary policy transmission process will gradually increase in the same way as in other countries of the euro area. By contrast, the importance of the exchange rate channel is going to decrease because Latvia's exports to and imports from the EMU countries will no longer be affected by the exchange rate fluctuations, while the share of these countries in Latvia's exports and imports will continue to grow. With Latvia's financial sector consolidating, the capital market will develop at a faster pace under the impact of, *inter alia*, enterprises increasing their financing on the capital market.

With Latvia moving towards integration with the EMU, the principles of the Bank of Latvia's monetary policy will more pronouncedly get in line with those of the ECB; with the Bank of Latvia becoming a member of the Eurosystem, the basic objective of its monetary policy will be subordinated to the ECB objective to keep inflation at 2% in the medium term. The formulation of this objective slightly differs from the one provided for the central bank under the Law "On the Bank of Latvia" which states: "The main objective of the Bank of Latvia shall be to implement monetary policy by controlling the amount of money in circulation with the aim to maintain price stability". The Bank of Latvia has been pursuing this goal using the peg of the lats to the SDR basket of currencies since 1994. Such strategy is based on serious considerations that have been explained by the Bank of Latvia in various official documents and analytical publications. In line with Latvia's accession to the EU and its economy getting integrated with those of the countries in the euro area, the lats will be pegged to the euro, whereas upon integration with the EMU, the euro will replace the lats and become the sole legal tender in Latvia. It will mark the end of the fixed exchange rate regime in Latvia, which simultaneously will be the end of the exchange rates as an interim target. The envisaged changes also indicate that alongside with the monetary policy objectives the entire strategy of the Bank of Latvia will be brought in line with the ECB strategy, with interest rates being the principal instrument affecting money supply and hence also the economic activity and inflation.

Substituting a floating exchange rate for the fixed one requires new monetary indicators for the assessment of the economy and the monetary policy, irrespective of a new interim target being set or not. The ECB whose monetary policy rests upon two pillars uses the implied forward rate alongside with several other indicators under the second pillar. Forward rates implied by the yield curve are used by the ECB mainly for the purpose of detecting whether they also contain interest rate, inflation and eco-

conomic activity expectations of market participants, thus supporting elucidation of the euro area's transmission mechanism, assessing market participants' confidence in the monetary policy and transparency of its own actions.

Since pegging the lats to the SDR basket of currencies, the Bank of Latvia's pursuit of an independent interest rate policy has, to a certain extent, been limited. In setting official interest rates, the Bank of Latvia has been and still is dependent on maintaining the fixed exchange rate under the impact of money market trends. Under such circumstances, the information contained in forward rates is unlikely to be a criterion of transparency for the activities of the central bank in a short term.

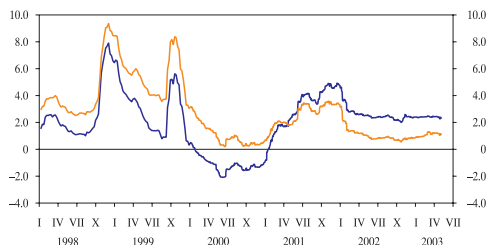
Under a fixed exchange rate regime, the exchange rate of the lats is obviously the central determinant of the money market interest rates over a longer horizon; daily fluctuations of the money market interest rates, on the other hand, do not necessarily depend on movements in the exchange rate but rather on the supply of and the demand for the lats, which in turn, are driven, for instance, by flows of the government resources in the banking system, changes in the volume of cash in circulation etc. It should be noted that the objective of the Bank of Latvia is not attainment of a certain interest rate but rather smoothing of excess interest rate fluctuations on the money market or intervention in interest rate processes when the exchange rate of the national currency is under pressure.

Of late, with interbank market activities and the Bank of Latvia open market operations expanding, inflation falling to the level of developed countries and macroeconomic situation stabilising, a larger impact of interest rates set by the Bank of Latvia on short-term (up to 1 month) money market interest rates is observed; on the other hand, since 2003, money market interest rates for a longer term (over 1 month) have been closer related to the SDR interest rates, particularly those of the US dollar, its component (see Chart 4.1).

Chart 4.1

**SPREADS BETWEEN 3-MONTH RIGIBOR AND LIBOR EUR AND RIGIBOR AND LIBOR USD (%)**

— Spread between RIGIBOR and LIBOR EUR  
 — Spread between RIGIBOR and LIBOR USD



Interest rates set by the Bank of Latvia on bank deposits with the Bank of Latvia and on Lombard loans form an interest rate corridor within which money market rates with a 7-day term fluctuate. Chart 4.2 shows that the Bank of Latvia repo rate becomes a benchmark for the domestic interbank market, while the spread of 3-month

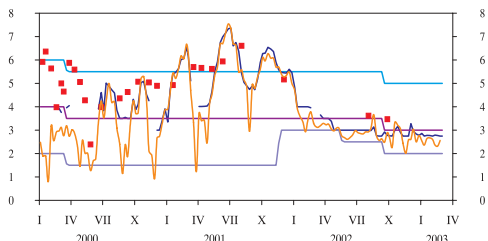


RIGIBOR and LIBOR for the US dollar has been comparatively stable since the beginning of 2002 (slightly over 2.0 percentage points).

Chart 4.2

**INTEREST RATES SET BY THE BANK OF LATVIA AND INTEREST RATES IN THE LATVIAN MONEY MARKET (%)**

- Refinancing rate
- Weighted average repo rate (7 days)
- Interbank rate on loans in lats (overnight)
- Interest rate on government securities (6 months)
- Interest rate on time deposits (7 days)
- Lombard rate (up to 10 days)



Thus, the limited effect of the Bank of Latvia's monetary policy on long-term interest rates is well confirmed. However, it is the long-term interest rate dynamics that reflects forecasts of the market participants in relation to inflation and macroeconomic development trends and hence also their reliance on lower inflation levels in the future. From these positions, the constant and consistent decline in inflation and long-term interest rates on deals in lats observed since 1993 indicates that the monetary policy of the Bank of Latvia has succeeded in attainment of its objective (price stability).

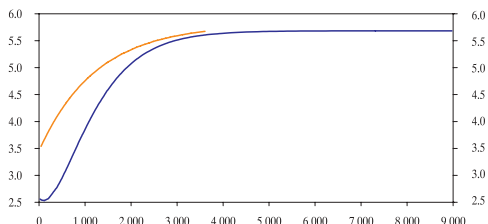
In the period separating us from integration with the EMU, Latvia's state risks and the one related to the exchange rate will further diminish, reducing the impact of risk premiums for assets in lats and the Bank of Latvia's interest rates on the national economy. It is supported by the studies on monetary policy transmission mechanisms conducted at the Bank of Latvia, which lead to a conclusion that interest rates of the euro area markets will become more important for the domestic economy, due to domestic and foreign assets becoming closer substitutes in the near future.(3)

The comparison of implied forward rates of the lats and the euro, and also the lats and the SDR is a vivid illustration for the conclusion above. The gap between short-term forward rates of the lats and the euro is gradually narrowing to 50 basis points over the horizon of 3–4 years and will almost disappear in 6–7 years (see Chart 4.3). With Latvia joining the euro area, the currency exchange risk of the Latvian govern-

Chart 4.3

**SHORT-TERM FORWARD RATES IN LATS AND EUROS (15.04.2003; in days; %)**

- In lats
- In euros

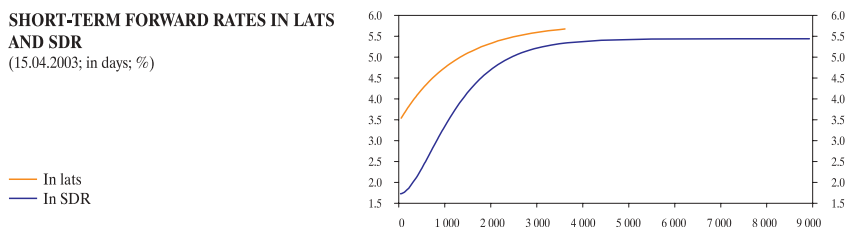


ment securities will disappear, and the interest rate differences between them and securities issued by the countries in the euro area could diminish at an even higher pace than implied by the forward rates. Already at this point, the interest rate gap between 10-year bonds of the Government of Latvia and respective bonds of the Government of Germany does not exceed 60 basis points.

On the other hand, the comparison of the forward rates of lats and the SDR (synthetic) demonstrates that on the whole forecasts of the SDR long-term interest rates are lower. Though for the last two years the money market longer term interest rates have been closer following the dynamics of the SDR rates, market participants expect the interest rate gap to persist over longer horizons mainly as a result of lower US dollar interest rate forecasts (see Chart 4.4).

Chart 4.4

**SHORT-TERM FORWARD RATES IN LATS AND SDR**  
(15.04.2003; in days; %)



Upon estimating the amount of risk premium incorporated in the interest rates of lats, it is essential to identify principal factors with an impact on it. At least four components of the risk premium can be identified:

$$RP = P_v + P_{vk} + P_i + P_l \tag{4.1}$$

- where  $RP$  is the risk premium;
- $P_v$  is the premium for state risk;
- $P_{vk}$  is the premium for currency exchange rate risk;
- $P_i$  is the premium for inflation;
- $P_l$  is the premium for liquidity.

For the purpose of this study, the risk premium is not specified, while application opportunities of yield curves may lead to an assumption that their comparison in various currencies may be useful in determining individual types of risks (see Chart 4.5). For example, when comparing yield curves of the euro area issuers' (governments in particular) instruments and those of the Latvian government eurobonds, it is possible to assess the state-risk-related component of the risk premium (a). As both the Latvian eurobonds and the financial instruments of the euro area are denominated in the same currency – the euro, the exchange rate risk of the two instruments is identical; the same is also true about the risk of the currency purchasing power. Assuming that the liquidity considerations for the two instruments do not differ notably, the only

distinguishing risk would be the state risk. To a great extent, the interest rate spread between the two instruments is influenced by Latvia's rating for long-term liabilities in foreign currencies. When, on the other hand, the comparison of the yield curves of the Latvian government eurobonds and the government securities issued in lats is made, the component of the risk premium depending on the impact of other risk factors, exchange rate and inflation risks as the most important ones, can be assessed (b). Charts 4.5 and 4.6 show respective spot rate curves of the euro money market, the Latvian eurobonds and government securities issued in lats.

Chart 4.5

**SPOT RATE CURVES FOR THE EURO MONEY MARKET, SECURITIES IN LATS AND EUROBONDS ISSUED BY THE LATVIAN GOVERNMENT**  
(21.01.2002; in days; %)

— Securities in lats  
— Eurobonds  
— Euro money market

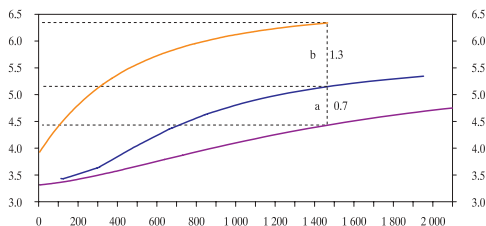
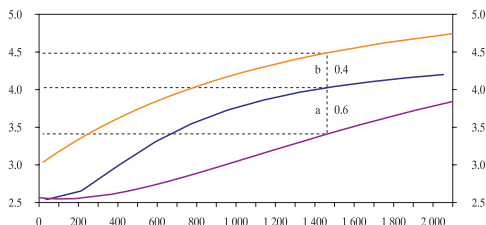


Chart 4.6

**SPOT RATE CURVES FOR THE EURO MONEY MARKET, SECURITIES IN LATS AND EUROBONDS ISSUED BY THE LATVIAN GOVERNMENT**  
(16.04.2003; in days; %)

— Securities in lats  
— Eurobonds  
— Euro money market



In comparison with January 2002, all interest rates declined and interest rate spreads narrowed in April 2003, indicating reduction in the respective risk factors. Favorable rating of Latvia by the international rating agency *Moody's Investors Service* in November 2002, when the latter raised Latvia's rating for long-term loans in foreign currencies from Baa2 to A2, fostered a decline in Latvia's state risk and hence also the narrowing of the spread between the interest rates of the Latvian eurobonds and the instruments of the euro area's issuers. This rise in rating was based on Latvia's gradual economic and financial integration with the EU and the invitation to join the EU and the NATO. It undeniably promoted reduction in the exchange rate risk for the Latvian government securities resulting in a narrowing interest rate spread between the latter and the euro instruments.

Upon joining the Eurosystem, the Bank of Latvia will have to make its research contribution to the attainment of common monetary goals of the euro area. As has been referred above, the process of obtaining and analysing information implied in the interest rate term structure plays an important part within the second pillar of the

ECB monetary policy. Common monetary policy of the euro area notwithstanding, central banks retain extensive linkages with the local money markets. Moreover, for the purpose of obtaining an overall picture of the euro area market participants' mood, the central bank of any country is able to provide a more accurate evaluation of its national market than a single centralized body, i.e. the ECB in this case. To use this information, studies of interconnections, if any, between forward rates and related economic variables are needed. In respect of Latvia's data, this study deals with inter-connection between forward rates implied in the yield curve and short-term interest rates in the future.

Holding of the interest rate expectations theory (though in its weaker form) allows forecasting of respective interest rates. As the empirical results obtained for the Latvian money and government securities markets for the interest rate expectations theory do not allow rejecting the possibility of using the yield curve in short-term interest rate forecasting, testing of the forecast quality seems useful. It can be performed by comparing the forecasts obtained with actual data at a certain period in the past using the so-called *out-of-sample* forecast quality test.

The essence of the method is as follows. First, the following regression model

$${}_{t+m}r_j - {}_t r_j = \beta_j ({}_t f_{j,t+m} - {}_t r_j) + \alpha_j \quad [4.2],$$

is considered using data at the time period from  $m_1$  to  $m_2$  (e.g. from February 1, 2002 to November 29, 2002 which corresponds to Period 3 of the money market study). As a result, the estimated forward rate values (corresponding to the expected interest rate values) are obtained:

$${}_{t+m}r_{progn,j} = \beta_j ({}_t f_{j,t+m} - {}_t r_j) + {}_t r_j + \alpha_j \quad [4.3].$$

The obtained  ${}_{t+m}r_{progn,j}$  estimations are sometimes treated as *in-sample* forecasts, i.e. these forecasts correspond to the time period from  $m_1$  to  $m_2$  (from February 1, 2002 to November 29, 2002). *Out-of-sample* forecasts, in turn, are calculated for the time period from  $(m_2 + 1)$  to  $m_3$ , i.e. the period for which actual interest rates are known. In such a way, the forecasts can be compared with the actual values, thus deriving estimates also for the model quality at the *out-of-sample* period. The mean absolute percentage error or MAPE can be used as a quantitative indicator of the forecast quality. The former is obtained using the following formula:

$$MAPE = \frac{1}{N} \sum_{t=m_2+1}^{m_3} \left| \frac{{}_{t+m}r_{progn,j} - {}_{t+m}r_j}{{}_{t+m}r_j} \right| \quad [4.4],$$

where  $N$  is the number of estimations;

${}_{t+m}r_{progn,j}$  are interest rate values expected for the period  $(t + m)$ ;

${}_{t+m}r_j$  are actual corresponding interest rate values for the period  $(t + m)$ .

Plausibility of forecasts can be tested by calculating the forecast plausibility interval, within which the estimated interest rate value lies. The larger the interval, the poorer is the accuracy of forecasts. The minimum and maximum band of the interval within which the interest rate forecast will fall with 95% probability is derived as follows:

$$(r_{progn} - 2 Error_{progn}; r_{progn} + 2 Error_{progn}) \quad [4.5],$$

where the standard error of expectations is  $Error_{progn} = S.E. \times \sqrt{1 + (s.e.(\beta))^2}$ , in which  $S.E.$  is the regression standard error and  $s.e.(\beta)$  is the standard error of the coefficient  $\beta$ .

Cases of *out-of-sample* expectations quality testing for 1- and 2-week interest rates of the money market are given in Charts 4.7 and 4.8.

On the basis of stationary results of time series, the interest rate expectations theory for the money market could be estimated for up to two months if daily time series were used, and three months if weekly time series were used. It is clear that plausibility intervals for various interest rates form corridors within the range of 20–60 basis points, which on average correspond to 7%–20% of the interest rate level, respectively. MAPEs indicated in the Charts are not perfectly comparable because the interest rate forecast horizons are different, and hence the number of estimations for a fixed expectations interval is different in each regression. Nevertheless, the comparison of the actual and expected interest rates given in Charts 4.7 and 4.8 provides an idea of forecast accuracy.

The MAPE for 1-week interest rate forecast for one week forward is 0.05. Obviously, the largest inconsistency between the actual and expected interest rates was observed at the end of 2002 and in early 2003, when the short-term money market interest rates fluctuated quite notably.

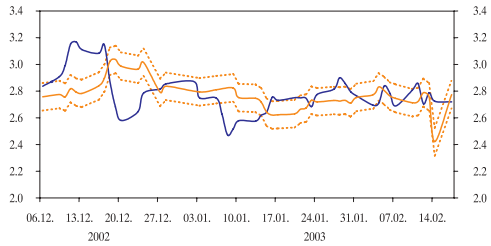
Such volatility of interest rates was determined by the largest demand for the lats experienced at the end of the year under the impact of seasonal factors when banks actively borrowed lats from the Bank of Latvia through repo and swap transactions. In January of 2003, by contrast, the demand for lats fell sharply, while redemption of the resources borrowed through repo transactions and currency swaps took the form of a gradual process. It determined surplus of liquid funds in the banking sector, which brought about a decline in interest rates. As spot rates for 1 and 2 weeks (see equation [1.11]), which usually change along with shifts in liquidity, are used in forecasting 1-week forward rates for one week forward, the former do not capture changes expected on the money market for a period that is longer than two weeks. 1 week's actual interest rate 1- and 2-month forward, on the other hand, fits in the forecast interval of changes. 1- and 2-month interest rates are more robust, i.e. they are not influenced by changes depending on short-term liquidity situation and are likely to capture the expected moves of the situation better than very short-term interest rates.

Chart 4.7

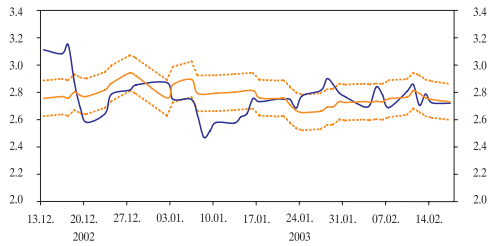
**FORECAST QUALITY TEST FOR 1-WEEK INTEREST RATE FOR THE MONEY MARKET**

(data time series 01.02.2002–29.11.2002 are used in forecasting;  $\pm 2 Error_{progn}$  is the interval incorporating interest rate forecast with 95% probability; %)

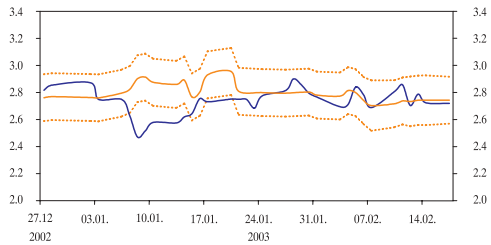
Forecasting horizon: 1 week forward  
MAPE = 0.0512



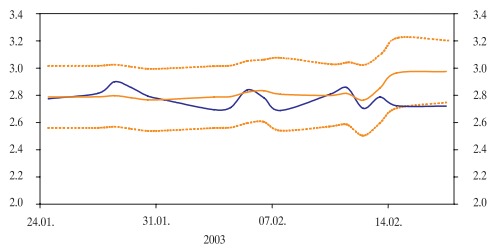
Forecasting horizon: 2 weeks forward  
MAPE = 0.0387



Forecasting horizon: 1 month forward  
MAPE = 0.0425



Forecasting horizon: 2 months forward  
MAPE = 0.0274



— Actual rate  
— Rate forecast  
- - -  $\pm 2 Error_{progn}$

2-week interest rate forecasting follows a similar trend, yet the results are better than those of the 1-week interest rate forecast, which, on the whole, is well founded because the 2-week interest rate is less volatile than the 1-week one.

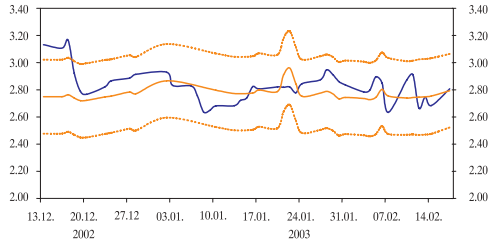
Further testing of the interest rate expectations theory is obviously needed in pursuit of a more detailed investigation of the informative content of also the long-term interest rates (over 1-year maturity) in respect of short-term interest rates. The results of stationarity tests in accord with respective selected testing periods obtained under

Chart 4.8

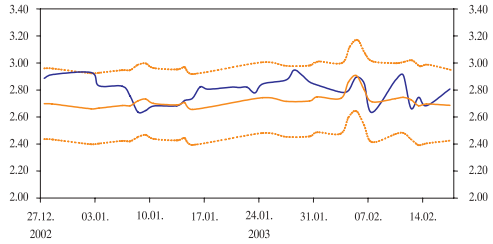
**FORECAST QUALITY TEST FOR 2-WEEK INTEREST RATE FOR THE MONEY MARKET**

(data time series 01.02.2002–29.11.2002 are used in forecasting;  $\pm 2 Error_{prog}$  is the interval incorporating interest rate forecast with 95% probability; %)

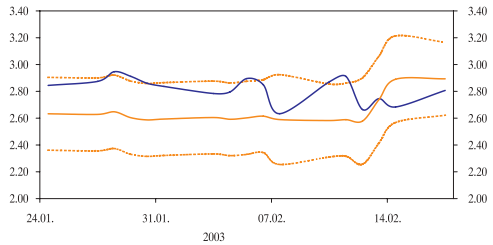
Forecasting horizon: 2 weeks forward  
MAPE = 0.0425



Forecasting horizon: 1 month forward  
MAPE = 0.0359

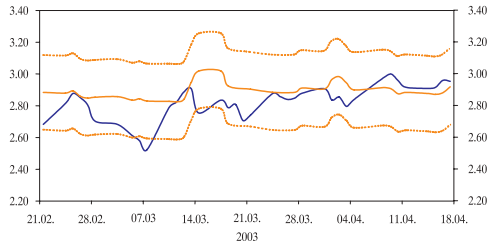


Forecasting horizon: 2 months forward  
MAPE = 0.0729



Forecasting horizon: 3 months forward  
MAPE = 0.0356

— Actual rate  
— Rate forecast  
- - -  $\pm 2 Error_{prog}$



this study allowed for examination of interest rates of only such a forecasting horizon that did not exceed 3 months, and only in exceptional cases was up to 1 year. It should be assumed that with the government securities market strengthening, liquidity of the secondary market and the informative content of prices will improve, the amount of the so-called irrational quotations will decrease promoting better quality of data time series and allowing for the application of analytical devices to the review of longer term interest rates and forecasting horizons. It is essential, however, that already now

the results of stationarity testing are very sensitive to the length of the selected period for a respective observation. The choice of a longer data time series period (e.g. 3 years) may increase the possibility of deriving stationary time series to a greater extent than may a shorter time period (more so when the shorter period is characterised by some particular pronounced trend of interest rate changes), thus allowing for the assessment of the informative content of longer term interest rates in relation to future short-term interest rates.

For example, when choosing the time period from April of 1998 to April of 2001, the forward rate dynamics for the forecast horizon up to 1 year can be assessed and interesting observations made (see Chart 4.9). The short-term forward rate dynamics is descending indicating that market participants were expecting a decline in short-term interest rates within the coming year, i.e. in the period from May of 2001 to April of 2002.

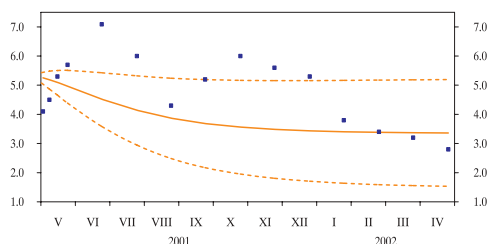
Attention should be drawn to the difference between the given interest rate forecast and forecast quality testing reflected in Charts 4.7 and 4.8. With the shorter horizon (up to 3 months) of the previous forecasts as an exception, each had a fixed forecast horizon. For example, 1-week interest rate forecast of one month forward was obtained from a respective regression where the 1-week forward rate ( $f_{7,t+30}$ ) estimated at the moment  $t$  for the forecast horizon ( $t + 30$ ) was used as a dependent variable. This forecast is based on the coefficient  $\beta$  obtained from the estimation of a respective regression and the assessment of the former's significance. The forecast of the short-term interest rate dynamics reflected in Chart 4.9, on the other hand, is based on the analysis of 14 regressions with similar interest rate terms (overnight transactions) but different forecast horizons (gradually growing from 1 week to 12 months).

The short-term forward rates thus obtained follow a descending trend. Chart 4.9 shows also the dynamics of actual interest rates on overnight loans for the period of the forecast. Despite the existing notable difference between the actual rates and the implied forward rates at some periods, overall tendencies for the period are the descending ones. This was a logical outcome of the descending interest rate term structure observed, for instance, on several occasions in the period of time between 1998

Chart 4.9

**SHORT-TERM INTEREST RATE FORECAST FOR 01.05.2001–01.05.2002 AND ITS 95% PLAUSIBILITY INTERVAL AND THE ACTUAL MONEY MARKET INTEREST RATES**  
 (Latvian government securities market data from 28.03.1998 to 01.05.2001 are used in forecasting; %)

- Rate forecast
- - - ±2 Error<sub>progn</sub>
- Average money market rate





and 2001 when the short-term interest rates on the money market were often higher than the long-term ones.

The period of data time series used in making forecasts is characterised by a notable instability of both the banking sector's liquidity and the interest rates (short-term ones in particular). By contrast, interest rates on longer-term transactions had a tendency to decline. That is why market participants occasionally perceived a rise in interest rates on short-term instruments of the money market as a short-term increase. For instance, the rise in short-term interest rates (lesser in long-term rates) reported in 2001 was to a large extent associated with the Bank of Latvia's interventions in foreign currency sales caused by seasonal factors and conducted at the close of 2000, as well as the placing in the domestic market of 5-year government bonds which impaired the banking sector's liquidity of the lats. At the end of 2001, on the other hand, when the Bank of Latvia engaged in foreign currency purchase activities, and with the Government's borrowing needs shrinking in 2002, money market short-term interest rates dropped significantly and became stable at the level of 3%–4%.

The fact that forward rates do not capture the actual short-term interest rate trend with adequate accuracy indicates that they are biased future short-term interest rate estimations, and as such they should be treated with a certain degree of circumspection. That is why further studies incorporating comprehensive analysis are needed to assess the time-varying risk premium. Application of GARCH (Generalised Autoregressive Conditional Heteroskedasticity) models is one of the approaches leading to a more accurate analysis of the interest rate forecast error, duly accounting for the dependence of conditional variance on time.

The existence of the risk premium appears to be the most serious drawback of using financial asset prices in the monetary policy. Further growth and deepening of the government securities market will improve data availability and possibilities to assess risk premiums more precisely. It is assumed that greater plausibility of the implied information content to a great extent depends on the future development of the financial market. If markets are not liquid, interpretation of these instruments is rendered more complicated because the search for the needed information requires application of additional technical assumptions.

## CONCLUSIONS

1. Since the 1990s, the interest rate term structure or the yield curve has been used as a standard instrument of the monetary policy decision-making process by central banks in developed countries. With Latvia moving towards integration with the EU and the EMU, the Bank of Latvia will have to revise its monetary policy and re-direct its research towards attainment of the monetary goals common for the euro area. Upon evaluating the impact of the monetary policy on the national economy, among other indicators the ECB relies on the assessment of a wide range of financial factors. Estimation of the interest rate term structure and its properties is dominant in the assessment of various financial instruments and market expectations.

2. To have a clear perception of the formation of the interest rate term structure on the Latvian financial market, its most acceptable functional form was first identified. The paper made an attempt to specify a method for yield curve modeling that would more accurately capture the data of both the Latvian money market and the government securities market, and would also be useful for the assessment of respective financial instruments and market participants' expectations for future interest rates. As a result, the Nelson–Siegel model was acknowledged as the most acceptable for the Latvian financial market.

3. Though the Svensson model (extended Nelson–Siegel model) is used by the ECB and the majority of EU central banks, the results obtained so far do not allow making conclusions about its usefulness for the Latvian monetary policy goals. Although the theoretical yield curve of the money market obtained by the Svensson model conforms to empirical data extremely well, in the majority of cases its results are quite unstable when the government securities market data are used.

4. There is an essential precondition for the yield curve to contain significant information on future interest rates, i.e. for the obtained forward rates to reflect actual future interest rates most accurately. This precondition is related to acceptance of the so-called interest rate expectations theory for a particular financial market. According to this theory, there is a close interrelation between short-term interest rates implied in the present long-term interest rates and expectations of the market participants for actual short-term interest rates in the future.

5. Results of testing the interest rate expectations theory on the Latvian financial market

5.1. The results of stationarity allowed for testing the interest rate expectations theory for the following maturities and forecast horizons: 7-, 14- and 30-day interest rate forecasts for up to a 3-month horizon for the Latvian money market, and 1- and 3-month interest rate forecasts for up to a 6-month horizon on the government securities market.

5.2. Rejection of the hypothesis about the existence of a zero risk premium on both financial market segments leads to an inference that forward rates in general (with a few exceptions that cannot be considered as an unequivocal result) do not ensure unbiased forecasts of spot rates, and the pure interest rate expectations theory cannot be applied in interest rate forecasting. Long-term interest rates contain a risk premium that is other than zero. This conforms well to the results obtained from the studies conducted on the financial markets of developed countries.

5.3. Without rejecting the hypothesis about the presence of a constant risk premium on the money market, it should be concluded that since April of 2000, forecasts of short-term interest rates (up to 30 days) for 3 months and on certain occasions also up to 1 year, can be made for the Latvian money market by applying the interest rate expectations theory. As to the securities market, the empirical results obtained differ, and the rejection of the hypothesis about a constant risk premium for more than a half of estimations implies that, for the time being, the interest rate expectations theory cannot be applied to the Latvian government securities market for the purpose of interest rate forecasting. Distinctive results obtained on the Latvian money and government securities markets can be associated with different liquidity and hence also with the quality of interest rates quoted. Of late, the money market has advanced substantially in terms of both its turnover and the range of instruments used, while on the government securities market, despite an increase in its volume and the range of maturities, the secondary market turnover is still moderate.

## APPENDICES

**STATISTICAL CRITERIA FOR DAILY YIELD CURVES OBTAINED FOR THE LATVIAN MONEY MARKET  
BY VARIOUS METHODS OF MODELING**

Model	RMSE	Quality by RMSE (1 is the best)	Smallest number of RMSE in period	$R^2$	Adjusted $R^2$
Period 1 (December 8, 1997–March 31, 2000)					
Polynomial model	0.290	5	4	0.953	0.922
Exponential spline model	0.149	2	70	0.986	0.932
Nelson–Siegel model	0.183	4	24	0.980	0.950
Vasicek model	0.183	4	3	0.980	0.967
Cox–Ingersoll–Ross model	0.180	3	2	0.981	0.953
Svensson model	0.086	1	485	0.996	–
Period 2 (April 1, 2000–January 31, 2002)					
Polynomial model	0.258	6	1	0.851	0.799
Exponential spline model	0.096	2	102	0.971	0.927
Nelson–Siegel model	0.122	3	11	0.940	0.894
Vasicek model	0.138	5	2	0.896	0.881
Cox–Ingersoll–Ross model	0.123	4	6	0.933	0.910
Svensson model	0.052	1	345	0.985	–
Period 3 (February 1, 2002–January 31, 2003)					
Polynomial model	0.104	6	0	0.943	0.935
Exponential spline model	0.036	2	73	0.990	0.977
Nelson–Siegel model	0.064	4	1	0.977	0.960
Vasicek model	0.070	5	1	0.966	0.965
Cox–Ingersoll–Ross model	0.053	3	0	0.979	0.973
Svensson model	0.023	1	177	0.996	0.985

**STATISTICAL CRITERIA FOR DAILY YIELD CURVES OBTAINED FOR THE LATVIAN GOVERNMENT SECURITIES MARKET BY VARIOUS METHODS OF MODELING**

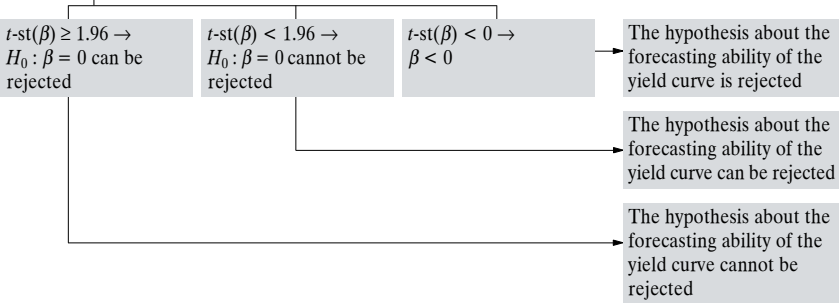
Model	RMSE	Quality by RMSE (1 is the best)	Smallest number of RMSE in period	$R^2$	Adjusted $R^2$
Period 1 (August 30, 1999–March 31, 2000)					
Polynomial model	1.127	6	1	0.342	0.333
Exponential spline model	0.809	4	20	0.669	0.612
Nelson–Siegel model	0.950	5	0	0.617	0.618
Vasicek model	0.508	3	0	0.842	0.827
Cox–Ingersoll–Ross model	0.487	2	1	0.852	0.819
Svensson model	0.339	1	131	0.858	0.836
Period 2 (April 1, 2000–January 31, 2002)					
Polynomial model	0.314	2	24	0.789	0.744
Exponential spline model	0.482	5	23	0.762	0.691
Nelson–Siegel model	0.375	6	6	0.775	0.686
Vasicek model	0.345	3	59	0.791	0.715
Cox–Ingersoll–Ross model	0.260	1	21	0.832	0.742
Svensson model	–	–	159	0.837	–
Period 3 (February 1, 2002–January 31, 2003)					
Polynomial model	0.259	1	0	0.845	0.783
Exponential spline model	0.305	2	73	0.768	0.709
Nelson–Siegel model	0.345	3	1	0.733	0.621
Vasicek model	0.381	4	1	0.724	0.615
Cox–Ingersoll–Ross model	0.426	5	0	0.670	0.637
Svensson model	–	6	177	0.780	0.634

**SPECIFICATION OF HYPOTHESES SELECTED FOR TESTING INTEREST RATE EXPECTATIONS THEORY**

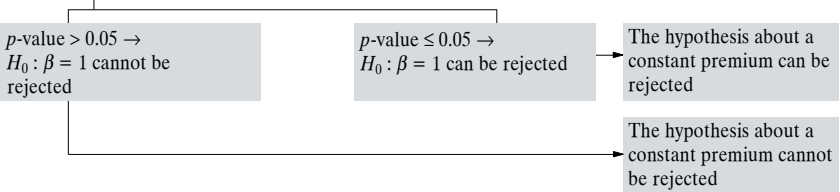
$${}_{t+m}r_j = \beta_j {}_t f_{j,t+m} + \alpha_j$$

$${}_{t+m}r_j - {}_t r_j = \beta_j ({}_t f_{j,t+m} - {}_t r_j) + \alpha_j$$

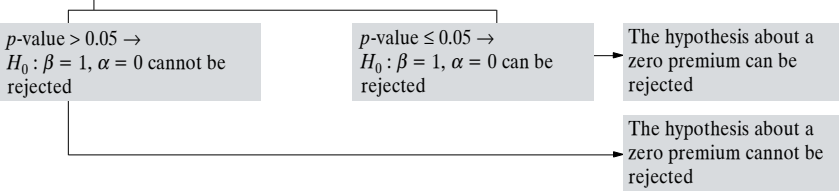
$H_0: \beta = 0$



$H_0: \beta = 1$



$H_0: \beta = 1, \alpha = 0$



## TESTING THE INTEREST RATE EXPECTATIONS THEORY FOR THE LATVIAN MONEY MARKET

No.	Dependent	Variable	$\beta$	$R^2$	Number of observations	$H_0: \beta = 0$ , $t$ -statistic	$Wald$ test $H_0: \beta = 1$ , $p$ -value	$Wald$ test $H_0: \beta = 1$ , $\alpha = 0$ , $p$ -value	Durbin-Watson statistical value
1	2	3	4	5	6	7	8	9	10
Period 1 (December 8, 1997–March 31, 2000) Daily time series									
1.	$r_{t-1}^{\gamma} - r_t^{\gamma}$	$r_t^{\gamma} - i_{t+7}^{\gamma}$	0.1438	0.0096	570	1.3897			
2.	$r_{t-1}^{\gamma} - r_{t+14}^{\gamma}$	$r_t^{\gamma} - i_{t+14}^{\gamma}$	0.1049	0.0038	564	0.6761			
3.	$r_{t-1}^{\gamma} - r_{t+30}^{\gamma}$	$r_t^{\gamma} - i_{t+30}^{\gamma}$	0.1156	0.0029	557	0.5707			
Weekly time series									
4.	$r_{t-1}^{\gamma} - r_t^{\gamma}$	$r_t^{\gamma} - i_{t+7}^{\gamma}$	0.0679	0.0023	120	0.4607			
5.	$r_{t-1}^{\gamma} - r_{t+14}^{\gamma}$	$r_t^{\gamma} - i_{t+14}^{\gamma}$	0.0066	0.0000	119	0.0257			
Period 2 (April 1, 2000–January 31, 2002) Daily time series									
6.	$r_{t-1}^{\gamma} - r_t^{\gamma}$	$r_t^{\gamma} - i_{t+7}^{\gamma}$	0.2416	0.0272	451	<b>2.0322</b>	<b>0.0000</b>		
7.	$r_{t-1}^{\gamma} - r_{t+14}^{\gamma}$	$r_t^{\gamma} - i_{t+14}^{\gamma}$	0.3419	0.0428	445	<b>2.8972</b>	<b>0.0000</b>		
8.	$r_{t-1}^{\gamma} - r_{t+30}^{\gamma}$	$r_t^{\gamma} - i_{t+30}^{\gamma}$	0.8051	0.1708	437	<b>4.9854</b>	0.2280	<b>0.0000</b>	0.0489
Weekly time series									
9.	$r_{t-1}^{\gamma} - r_{t+14}^{\gamma}$	$r_t^{\gamma} - i_{t+14}^{\gamma}$	0.2917	0.0363	94	<b>1.9648</b>			

$t$ -statistic and  $p$ -value, permitting rejection of a respective hypothesis under the 95% significance, are indicated in bold.



No.	Dependent	Variable	$\beta$	$R^2$	Number of observations	$H_0: \beta = 0$ , $t$ -statistic	Wald test $H_0: \beta = 1$ , $p$ -value	Wald test $H_0: \beta = 1$ , $\alpha = 0$ , $p$ -value	Durbin-Watson statistical value
1	2	3	4	5	6	7	8	9	10
Period 3 (February 1, 2002–November 29, 2002)									
Daily time series									
10.	$i^T7 - i + i^T7$	$i^T7 - i^T7_{i+7}$	1.1918	0.1484	199	<b>5.8583</b>	0.3470	<b>0.0000</b>	0.3938
11.	$i^T7 - i + 14i^T7$	$i^T7 - i^T7_{i+14}$	1.9309	0.3376	195	<b>3.9374</b>	0.0592	<b>0.0000</b>	0.4357
12.	$i^T7 - i + 30i^T7$	$i^T7 - i^T7_{i+30}$	1.3956	0.3521	185	<b>5.1392</b>	0.1469	<b>0.0000</b>	0.3455
13.	$i^T7 - i + 60i^T7$	$i^T7 - i^T7_{i+60}$	1.2128	0.3603	166	<b>4.7810</b>	0.4027	<b>0.0000</b>	0.3000
14.	$i^T14 - i + i^T14$	$i^T14 - i^T14_{i+7}$	1.0259	0.1472	199	<b>2.7360</b>	0.9450	<b>0.0000</b>	0.4203
15.	$i^T14 - i + 14i^T14$	$i^T14 - i^T14_{i+14}$	1.6906	0.3553	195	<b>4.6084</b>	0.0613	<b>0.0000</b>	0.4623
16.	$i^T14 - i + 30i^T14$	$i^T14 - i^T14_{i+30}$	1.3087	0.3608	190	<b>5.5840</b>	0.1894	<b>0.0000</b>	0.3810
17.	$i^T14 - i + 60i^T14$	$i^T14 - i^T14_{i+60}$	1.2229	0.3792	166	<b>5.1027</b>	0.3537	<b>0.0000</b>	0.3223
18.	$i^T30 - i + 30i^T30$	$i^T30 - i^T30_{i+30}$	1.0917	0.3700	185	<b>5.9692</b>	0.6166	<b>0.0000</b>	0.4466
19.	$i^T30 - i + 60i^T30$	$i^T30 - i^T30_{i+60}$	1.1444	0.4257	166	<b>5.7114</b>	0.4722	<b>0.0000</b>	0.3570
Weekly time series									
20.	$i + i^T7$	$i^T7_{i+7}$	0.9790	0.8785	44	<b>17.4232</b>	0.7105	<b>0.0000</b>	1.6494
21.	$i + 14i^T7$	$i^T7_{i+14}$	0.9767	0.8493	44	<b>20.6218</b>	0.6248	<b>0.0000</b>	1.5305
22.	$i^T7 - i + 30i^T7$	$i^T7 - i^T7_{i+30}$	1.5676	0.3789	40	<b>4.7658</b>	0.0925	<b>0.0000</b>	0.9646
23.	$i^T7 - i + 60i^T7$	$i^T7 - i^T7_{i+60}$	1.3323	0.3889	36	<b>4.1775</b>	0.3048	<b>0.0000</b>	1.0626
24.	$i^T7 - i + 90i^T7$	$i^T7 - i^T7_{i+90}$	1.2283	0.4425	32	<b>4.6283</b>	0.3964	<b>0.0000</b>	0.9345
25.	$i^T14 - i + i^T14$	$i^T14 - i^T14_{i+7}$	0.8063	0.0899	43	<b>2.3714</b>	0.5719	<b>0.0000</b>	2.2218

$t$ -statistic and  $p$ -value, permitting rejection of a respective hypothesis under the 95% significance, are indicated in bold.

(Appendix 4 cont.)

No.	Dependent	Variable	$\beta$	$R^2$	Number of observations	$H_0: \beta = 0$ , t-statistic	Wald test $H_0: \beta = 1$ , p-value	Wald test $H_0: \beta = 1$ , $\alpha = 0$ , p-value	Durbin-Watson statistical value
1	2	3	4	5	6	7	8	9	10
26.	$r_{14}^- - t + 14r_{14}$	$r_{14}^- - f_{14,t} + 14$	2.0110	0.4297	42	<b>5.4901</b>	<b>0.0087</b>		
27.	$r_{14}^- - t + 30r_{14}$	$r_{14}^- - f_{14,t} + 30$	1.3307	0.3565	40	<b>4.8787</b>	0.2328	<b>0.0000</b>	1.0604
28.	$r_{14}^- - t + 60r_{14}$	$r_{14}^- - f_{14,t} + 60$	1.2423	0.3693	36	<b>3.9733</b>	0.4437	<b>0.0000</b>	1.1416
29.	$r_{14}^- - t + 90r_{14}$	$r_{14}^- - f_{14,t} + 90$	1.2970	0.9394	32	<b>4.9292</b>	0.2679	<b>0.0000</b>	0.9394
30.	$r_{30}^- - t + 30r_{30}$	$r_{30}^- - f_{30,t} + 30$	1.0072	0.3275	40	<b>4.9138</b>	0.9721	<b>0.0000</b>	1.1332
31.	$r_{30}^- - t + 60r_{30}$	$r_{30}^- - f_{30,t} + 60$	1.1340	0.3906	36	<b>4.4363</b>	0.6035	<b>0.0000</b>	1.2464
32.	$r_{30}^- - t + 90r_{30}$	$r_{30}^- - f_{30,t} + 90$	1.2633	0.6000	32	<b>5.5402</b>	0.2574	<b>0.0000</b>	0.9970

t-statistic and p-value, permitting rejection of a respective hypothesis under the 95% significance, are indicated in bold.

## TESTING THE INTEREST RATE EXPECTATIONS THEORY FOR THE LATVIAN GOVERNMENT SECURITIES MARKET

No.	Dependent	Variable	$\beta$	$R^2$	Number of observations	$H_0: \beta = 0$ , $t$ -statistic	Wald test $H_0: \beta = 1$ , $p$ -value	Wald test $H_0: \beta = 1$ , $\alpha = 0$ , $p$ -value	Durbin-Watson statistical value
1	2	3	4	5	6	7	8	9	10
Period 1 (August 30, 1999–March 31, 2000) Daily time series									
1.	$r_{30}^{t-1} + 30r_{30}^{t-30}$	$r_{30}^{t-1} - r_{30}^{t-30}$	0.7996	0.1326	62	1.7456			
Period 2 (April 1, 2000–October 31, 2001) Daily time series									
2.	$r_{30}^{t-1} + 30r_{30}^{t-30}$	$r_{30}^{t-1} - r_{30}^{t-30}$	1.8199	0.1616	244	<b>2.4590</b>	0.2691	0.1850	0.3766
3.	$r_{30}^{t-1} + 60r_{30}^{t-60}$	$r_{30}^{t-1} - r_{30}^{t-60}$	-0.2566	0.0049	243	-0.3614			
4.	$r_{30}^{t-1} + 90r_{30}^{t-90}$	$r_{30}^{t-1} - r_{30}^{t-90}$	2.5637	0.3683	222	<b>4.6677</b>	<b>0.0048</b>		
5.	$r_{30}^{t-1} + 180r_{30}^{t-180}$	$r_{30}^{t-1} - r_{30}^{t-180}$	0.4985	0.1749	178	<b>2.6416</b>	<b>0.0086</b>	0.8744	0.2777
6.	$r_{90}^{t-1} + 30r_{90}^{t-30}$	$r_{90}^{t-1} - r_{90}^{t-30}$	1.0705	0.1530	244	<b>2.4015</b>	0.8744	<b>0.0000</b>	
7.	$r_{90}^{t-1} + 60r_{90}^{t-60}$	$r_{90}^{t-1} - r_{90}^{t-60}$	-0.4717	0.0382	243	-0.9891			
8.	$r_{90}^{t-1} + 90r_{90}^{t-90}$	$r_{90}^{t-1} - r_{90}^{t-90}$	1.5211	0.3131	222	<b>3.9909</b>	0.1730	0.0932	0.1528
9.	$r_{90}^{t-1} + 180r_{90}^{t-180}$	$r_{90}^{t-1} - r_{90}^{t-180}$	0.2285	0.0499	178	1.1783			
Weekly time series									
10.	$r_{30}^{t-1} + 30r_{30}^{t-30}$	$r_{30}^{t-1} + 30r_{30}^{t-30}$	1.1005	0.9557	75	<b>36.6186</b>	<b>0.0013</b>		
11.	$r_{30}^{t-1} + 60r_{30}^{t-60}$	$r_{30}^{t-1} + 60r_{30}^{t-60}$	1.1373	0.9223	75	<b>30.1089</b>	<b>0.0005</b>		

$t$ -statistic and  $p$ -value, permitting rejection of a respective hypothesis under the 95% significance, are indicated in bold.

(Appendix 5 cont.)

No.	Dependent	Variable	$\beta$	$R^2$	Number of observations	$H_0: \beta = 0$ , $t$ -statistic	Wald test $H_0: \beta = 1$ , $p$ -value	Wald test $H_0: \beta = 1$ , $\alpha = 0$ , $p$ -value	Durbin-Watson statistical value
1	2	3	4	5	6	7	8	9	10
12.	$t + 90^{\circ}30$	$f_{30,t+90}$	1.1679	0.8838	75	<b>26.2651</b>	<b>0.0003</b>	<b>0.0000</b>	0.7922
13.	$t + 180^{\circ}30$	$f_{30,t+180}$	1.2120	0.7153	75	<b>8.9327</b>	0.1225	<b>0.0000</b>	
14.	$t + 30^{\circ}90$	$f_{90,t+30}$	1.1353	0.8829	75	<b>17.8723</b>	<b>0.0366</b>	<b>0.0000</b>	
15.	$t + 60^{\circ}90$	$f_{90,t+60}$	1.1443	0.8348	75	<b>11.6865</b>	0.1450	<b>0.0000</b>	0.6832
16.	$t + 90^{\circ}90$	$f_{90,t+90}$	1.1438	0.7817	75	<b>8.2306</b>	0.3043	<b>0.0000</b>	0.6193
17.	$t + 180^{\circ}90$	$f_{90,t+180}$	1.0749	0.5776	75	<b>3.5926</b>	0.8029	<b>0.0000</b>	0.4717
Period 3 (October 1, 2001–September 30, 2002) <sup>1</sup>									
Daily time series									
18.	$t^{\circ}30 - t + 30^{\circ}30$	$t^{\circ}30 - f_{30,t+30}$	1.5187	0.2513	215	<b>5.1695</b>	0.0789	<b>0.0000</b>	0.4940
19.	$t^{\circ}30 - t + 60^{\circ}30$	$t^{\circ}30 - f_{30,t+60}$	1.7975	0.3803	195	<b>6.0311</b>	<b>0.0081</b>		
20.	$t^{\circ}30 - t + 90^{\circ}30$	$t^{\circ}30 - f_{30,t+90}$	2.1970	0.5805	177	<b>10.3633</b>	<b>0.0000</b>	<b>0.0000</b>	
21.	$t^{\circ}30 - t + 180^{\circ}30$	$t^{\circ}30 - f_{30,t+180}$	3.3753	0.6333	118	<b>8.4545</b>	<b>0.0000</b>	<b>0.0000</b>	0.3174
22.	$t^{\circ}90 - t + 30^{\circ}90$	$t^{\circ}90 - f_{90,t+30}$	0.7352	0.1945	215	<b>4.0720</b>	0.1439	<b>0.0000</b>	0.1716
23.	$t^{\circ}90 - t + 60^{\circ}90$	$t^{\circ}90 - f_{90,t+60}$	1.1595	0.3154	195	<b>4.7565</b>	0.5137	<b>0.0000</b>	
24.	$t^{\circ}90 - t + 90^{\circ}90$	$t^{\circ}90 - f_{90,t+90}$	1.6921	0.5389	177	<b>8.7181</b>	<b>0.0005</b>	<b>0.0000</b>	
25.	$t^{\circ}90 - t + 180^{\circ}90$	$t^{\circ}90 - f_{90,t+180}$	2.9957	0.7339	118	<b>8.5177</b>	<b>0.0000</b>	<b>0.0000</b>	
Weekly time series									
26.	$t^{\circ}30 - t + 30^{\circ}30$	$t^{\circ}30 - f_{30,t+30}$	1.5868	0.2343	49	<b>2.6189</b>	0.3378	<b>0.0000</b>	0.6502

$t$ -statistic and  $p$ -value, permitting rejection of a respective hypothesis under the 95% significance, are indicated in bold.

<sup>1</sup> Periods 2 and 3 partly overlap due to stationarity results.

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