LATVIJAS BANKA

A FACTOR MODEL OF THE TERM STRUCTURE OF INTEREST RATES AND RISK PREMIUM ESTIMATION FOR LATVIA'S MONEY MARKET

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Fragments of the painting *White Move* (2005) by Juta Policja and Mareks Gureckis have been used in the design.

CONTENTS

ABBREVIATIONS

CB – central bank
EONIA – Euro Overnight Index Average
GDP – gross domestic product
LIBOR – London Interbank Offered Rate
RIGIBID – Riga Interbank Bid Rate
RIGIBOR – Riga Interbank Offered Rate
SDR – Special Drawing Rights
VAR – vector autoregression

ABSTRACT

The paper presents the analysis of risk premium of the interest rate term structure for the Latvian money market. On the back of the approach used by F. Diebold, G. Rudebusch and B. Aruoba, it has been assumed that the coefficients of the Nelson–Siegel model are unobservable therefore the model of this research paper has been estimated using the Kalman filter. The risk premium behaviour has been obtained for interest rates of different maturities and forecasting horizons between May 2000 and July 2005. The results obtained indicate that the amount of the risk premium was significant and its volatility substantial between 2000 and 2002. In post-2002 period, its behaviour gradually stabilised and was marked by a downward trend after 2004.

Key words: *term structure of interest rates, risk premium, the Nelson–Siegel model, the Kalman filter*

JEL classification codes: *C32, D84, E43, E47, G10*

INTRODUCTION

Information captured in the prices of financial assets gives signals to CBs about the expectations of market participants regarding such fundamentals as the future economic activity, inflation, and short-term interest rate dynamics. The analysis of these expectations plays a significant role for the future policy process.

Aiming for its core objective of price stability, the Eurosystem, as an example, consistently adheres to the two-pillar strategy when implementing its monetary policy, and the financial asset prices are important for it as the second pillar indicator.(19)

Financial asset prices are a reflection of market participants' expectations because the former are, in fact, forward-looking. The present asset prices are determined by discounting expected future payment flows. Two factors affect the discount rate used in the financial asset assessment:
1) compensation for consumption postponed to the future and not used at the current point in time; and
2) compensation for the risk associated with the future payment flow uncertainties.

In the assessment of a financial asset, the investor shall be able to predict the future payment flows and the discount rates with risk premiums included that are applicable to these flows.

The price of fixed income financial instruments is determined by interest rates to be used in discounting the respective payment flows. The interest rates, in turn, are dependent upon the expectations of the fundamental macroeconomic variables like inflation and the real interest rates, as well as upon the compensation for risks related to uncertainty of the respective expectations.

Information about financial market participants' expectations regarding future interest rates is particularly significant for CBs because it helps them foresee if a particular decision will surprise market participants and what their short-term reaction to it could be. The official future interest rate expectations figure prominent also when the current monetary policy stance is formulated. Changes in long-term interest rates that primarily depend on the expected official future interest rates affect many participants of the financial market. Therefore, aiming for the assessment and control over current shifts in the monetary situation, CBs need to have some understanding about market participants' expectations about the official future interest rates.

Forward rates are the most widely used measure of interest rate expectations. They are the implied future interest rates incorporated in the present interest rates for different maturities. Provided that uncertainties associated with future interest rates were absent, forward interest rates would be equal to the expected future interest rates. However, the future interest rates are not known for sure. In order to hold this interest rate risk, the investors who want to avoid the risk would demand a risk premium. In such a way in equilibrium, this will drive a wedge – the risk premium – between the forward rate and the expected short-term interest rate. In addition, longer future horizons are related to a more pronounced uncertainty regarding possible interest rate movements, hence the respective risk premium is likely to increase with maturity. Consequently, the longer the time horizon, the larger becomes the difference between the forward rate and the expected rate.

In this study, the risk premium is defined as the difference between the forward rate and the expected future interest rate:

$$\mathrm{pr} = f - E_t i \,.$$

To estimate the expected future interest rate, the procedure proposed by F. Diebold and C. Li was used.(4) These authors proved that a relatively precise forecast of the term structure of interest rates can be derived from autoregressive models for factors corresponding to the level, slope and curvature of the yield curve. Basing on the approach of F. Diebold, G. Rudebusch and B. Aruoba (6), the study assumes that these unobservable factors correspond to the Nelson–Siegel model coefficients that are estimated and predicted in this study using the Kalman filter.(13)

The authors of this study have opted for the Kalman filter because it has certain advantages over other econometric methods. Due to the ongoing transition of Latvia's economy, a great number of economic variables are not stationary. As is known, the Kalman filter provides an opportunity to work with non-stationary variables. Moreover, economic variables are affected by a number of factors, e.g. the investment and political climate, that cannot be accurately estimated, and the Kalman filter allows for the estimation of economic variables and factors that are changing over time.

Chapter 1 defines some basic theoretical concepts that are needed for further analysis and builds the theoretical framework for the factor model of the term structure of interest rates. Chapter 2 deals with the factor model on the back of Latvia's data. Section 2.1 reviews the selected data sample. Section 2.2 analyses the estimates obtained by the Kalman filter. Section 2.3 presents the results of the estimated risk premium. Section 2.4 describes the empirical results. The most important effects of the study are summed up in the Conclusion. Appendix 1 presents the theoretical framework of the Kalman filter. Appendices 2–5 furnish the risk premium dynamics for 1-, 3-, 6- and 12-month interest rates at different forecasting horizons, while Appendix 6 sums up mean premiums and standard deviations for 1-, 3-, 6- and 12-month interest rates.

1. FACTOR MODELS OF THE TERM STRUCTURE OF INTEREST RATES

The introductory part of the Chapter defines and describes the basic theoretical concepts.

$i(t,T)$ denotes the nominal spot interest rate, i.e. the yield to maturity of a zero-coupon bond bought at time t with $T > t$ maturity date. Assuming that no-arbitrage restriction is imposed, the nominal implied forward interest rate at time t with the delivery term τ and maturity at T can be defined as follows (17):

$$f(t,\tau,T) = \frac{i(t,T)\cdot(T-t) - i(t,\tau)\cdot(\tau-t)}{(T-\tau)\cdot(1+i(t,\tau)\cdot(\tau-t))} \quad [1.1].$$

The forward rate premium is calculated as the difference between the forward interest rate and the expected future interest rate:

$$\mathrm{pr}(t,\tau,T) = f(t,\tau,T) - E_t i(\tau,T) \quad [1.2]$$

where E_t is the conditional mathematical expectations operator for information available at time t.

Taking into account that the forward rate at each time t can be calculated using equation [1.1], premium determination should rest upon the estimation of $E_t i(\tau,T)$. Provided that an appropriate model is used, this term can be defined as a modelled forecast for the respective interest rate. Obviously, the forecasts of different time horizons $\tau - t$ and those of interest rates on different maturities $T - \tau$ must be interrelated. Indeed, this model should capture the entire term structure of interest rates.

As there are considerably fewer sources of systematic risk than there are tradable financial instruments, all price information of tradable interest-rate-based financial instruments can be accumulated in a few variables or factors.(15; 16) Consequently, term structure factor models of interest rates use structures with a small number of interest rate factors and the associated factor loadings that relate interest rates on different maturities to these factors. Factor structures ensure useful data compression and simultaneously the so-called parsimony principle.

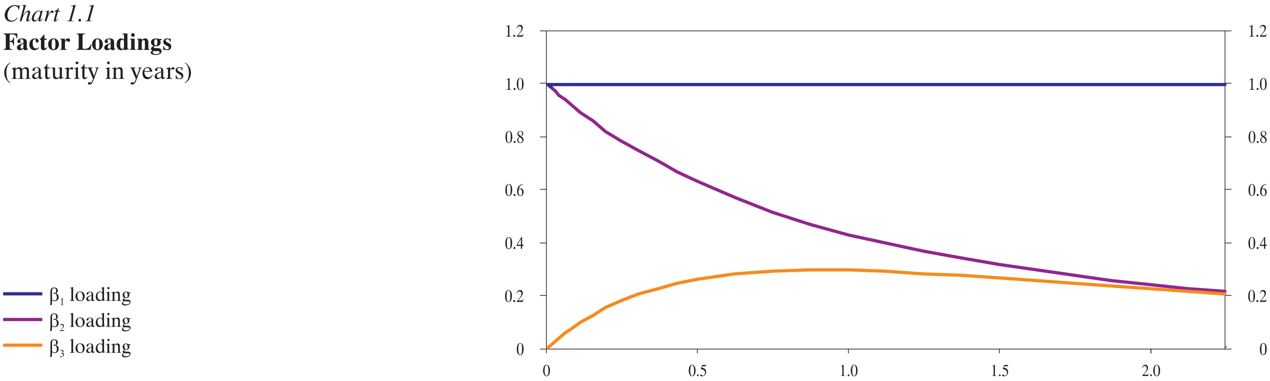
A number of approaches to constructing the interest rate factors and loadings of these factors are presented in the literature. The factors may be the first principal components that are orthogonal to each other by definition, while the loadings may be relatively unrestricted.(15; 16) The first three principal components are usually correlated to the level, slope and curvature of the yield curve. Another approach widely employed by practitioners and CBs is the Nelson–Siegel model (introduced in the work of C. Nelson and A. Siegel (17)). Factor loadings in the Nelson–Siegel model have economically plausible restrictions: the forward rates are always positive and, with the term to maturity increasing, the discount function approaches zero. A no-arbitrage dynamic latent factor model is the third approach. The most general subclass of latent factor models postulates a linear or affine functional relationship of latent factors with interest rates and restrictions on the factor loadings that rule out arbitrage strategies involving interest rate instruments.

According to the factor model approach, a large set of yields of various maturities is expressed as a function of a small number of unobserved factors. The set of yields is denoted as $y(\tau)$ where τ is the term to maturity. CBs use widely the Nelson-Siegel (17) curve to represent the cross-section yield data:

$$y(\tau) = \beta_1 + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad [1.3]$$

where β_1, β_2, β_3 and λ are the parameters. Parameter λ captures velocity at which exponential terms decrease. The study assumes it to be a constant because this assumption considerably reduces volatility of parameters β_{it}, rendering their behaviour more predictable. Parameters β_{1t}, β_{2t} and β_{3t} are interpreted as three latent (unobservable) factors.(6) Factor loading of β_{1t} is equal to *1*, i.e. as $\tau \to \infty$, it remains unchanged, thus β_{1t} can be considered a long-term factor. Factor loading for β_{2t} is $\frac{(1-e^{-\lambda\tau})}{\lambda\tau}$. This function, equal to *1* if $\tau = 0$ and monotonically decreasing to *0*, can be considered a short-term factor. The loading of β_{3t} is $\frac{(1-e^{-\lambda\tau})}{\lambda\tau} - e^{-\lambda\tau}$; it is the function, equal to *0*, if $\tau = 0$ (i.e. it is not a short-term factor), growing to its maximum at $\tau \approx \frac{1.8}{\lambda}$, and afterward reversing to *0* (i.e. it is not a long-term factor) that hence can be considered a medium-term factor. Chart 1.1 presents the given factor loadings under the condition that $\lambda = 2$.

Chart 1.1
Factor Loadings
(maturity in years)



The long-term, short-term and medium-term factor can be interpreted as the level, slope and curvature of the yield curve, respectively. For example, the long-term factor β_{1t} describes the level of the yield curve. In addition, the relation $y(\infty) = \lim_{\tau\to\infty} y(\tau) = \beta_{1t}$ can be derived from equation [1.3]. An increase in β_{1t} would cause a rise by the same amount in all yields $y(\tau)$ and simultaneously push up the level of the yield curve.

Several authors, e.g. J. A. Frankel and C. Lown (11), define the slope of the curve as $y_t(\infty) - y_t(0)$, which, according to formula [1.3], is β_{2t}. In such a way, the short-term factor β_{2t} determines the slope of the yield curve. Due to an increase in β_{2t}, the short-term rates grow faster than the long-term rates because loading $\frac{(1-e^{-\lambda_t\tau})}{\lambda\tau}$, which is multiplied by β_{2t} at small τ values, would be close to *1*, but at large τ values it would be close to *0*, which, in turn, causes shifts in the slope of the yield curve.

An increase in factor β_{3t}, on the other hand, has little effect on the rise of short-term and long-term interest rates because factor loading $\frac{(1-e^{-\lambda_t\tau})}{\lambda\tau}-e^{-\lambda_t\tau}$ would be close to *0* at both large and small values of τ, and would more affect the growth of the medium-term interest rates (with a maximum growth in interest rates corresponding to maturity $\tau \approx \frac{1.8}{\lambda}$), thus increasing the curvature of the yield curve.

As has been proved by F. Diebold and C. Li (4), the Nelson–Siegel curve can be represented as a dynamic latent factor model with β_1, β_2 and β_3 as time-varying factors of level, slope and curvature; the terms multiplied by these factors are factor loadings. Thus the model can be represented as follows:

$$y_t(\tau) = L_t + S_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + C_t\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \qquad [1.4]$$

where L_t, S_t and C_t are the time-varying variables β_1, β_2 and β_3. This approach will be further supported by empirical estimates.

If the dynamics of L_t, S_t and C_t follows a vector autoregressive process of the first order, this model forms a state-space system. The transition equation governing the dynamics of the state vector is

$$\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} = \begin{pmatrix} \mu_L \\ \mu_S \\ \mu_C \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \end{pmatrix} + \begin{pmatrix} \eta_t(L) \\ \eta_t(S) \\ \eta_t(C) \end{pmatrix} \qquad [1.5]$$

where t = 1, …, T is the time series length in the sample. The equation that relates a set of N yields to the three unobservable factors can be written

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{pmatrix}\begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \vdots \\ \varepsilon_t(\tau_N) \end{pmatrix} \qquad [1.6].$$

Using generally accepted vector and matrix notations, the given state-space system can be written

$$\alpha_t = \mu + A\alpha_{t-1} + \eta_t \qquad [1.7],$$

$$y_t = \Lambda\alpha_t + \varepsilon_t \qquad [1.8],$$

where vector $\alpha_t = (L_t, S_t, C_t)'$.

To achieve the linear least squares optimality of the Kalman filter, we assume a condition that the white noise transition and measurement disturbances are orthogonal both mutually and relative to the initial state:

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} H & 0 \\ 0 & Q \end{pmatrix} \right] \qquad [1.9],$$

$$E(\alpha_0 \eta_t^{'}) = 0 \qquad [1.10],$$

$$E(\alpha_0 \varepsilon_t^{'}) = 0 \qquad [1.11].$$

The analysis is dominated by the assumption that H and Q matrices are diagonal. The assumption regarding a diagonal Q matrix, which implies mutually uncorrelated deviations of yields of various maturities from the yield curve, is quite common. Despite the fact that F. Diebold, G. Rudebusch and B. Aruoba did not impose any restrictions on matrix H, non-diagonal matrix elements turned out to be insignificant.(6) Therefore for computation simplicity, this paper deals only with the diagonal type of matrix H.

Overall, the state-space approach ensures an effective framework for the analysis and estimation of dynamic models. The inference that the Nelson–Siegel model can easily be transformed into a state-space model is particularly useful, because in this case the Kalman filter produces estimates of the maximum likelihood along with optimally filtered and smoothed estimates of the model factors. Moreover, in this paper preference is given to the one-step Kalman filter method rather than the two-step Diebold–Li approach because, according to the standard theory, simultaneous estimation of all parameters results in correct inferences. By contrast, the two-step approach has a drawback: the uncertainty of parameter estimation and signal extraction via the first step is not accounted in the second step computations. Furthermore, the state-space approach raises a possibility of future extensions, e.g. existence of heteroskedasticity, shortage of data, etc, albeit the present paper does not take on the task of dealing with such extensions.

It is useful to compare the approach used in this paper with those proposed by other authors. An unrestricted VAR estimated for a set of yields is a very general (linear) model. This model has a potential drawback of its results being possibly dependent on the particular selected set of yields. The aforementioned factor representation can aggregate information from a large set of yields. Another factor model ranking among the simplest ones is VAR estimated with the principal components[1], which have been formed from a large set of yields. This approach imposes a restriction on factors to be mutually orthogonal, yet it does not fully restrict factor loadings. However, the model used in this paper potentially allows for factor correlation but restricts factor loadings imposing limits on the set of admissible yield curves. For instance, the Nelson–Siegel

[1] VAR term structur analysis can be found in the works of e.g. C. Evans and D. Marshall (9; 10).

model guarantees positive forward interest rates for all time periods, and also, as maturities increase, the converging of the discount function toward *0*. Such economically-founded restrictions are likely to support the analysis of the yield curve dynamics. It is also possible to introduce alternative restrictions, of which the non-arbitrage restriction is imposed most often. It ensures consistency in interest rate adjustments of the yield curve over time. Nevertheless, the evidence on the extent to what these restrictions affect the results is quite varied.[2]

[2] Works of e.g. A. Ang and M. Piazessi (1) as well as G. Duffee (7) can be used for comparison.

2. EMPIRICAL RESULTS

2.1 Data

Arithmetic means of 1-, 3-, 6-, 9- and 12-month RIGIBID and RIGIBOR have been used to estimate the model in this study. Interest rates of shorter maturities are not analysed, because their variance (particularly that of overnight rates) is excessively pronounced at the end of the reserve maintenance period. Monthly data for the period from May 2000 to July 2005 have been used.[3] Interest rates have been computed as daily arithmetic means of the respective month.

2.2 Model Estimation

The model of this paper forms a state-space system where the VAR(1) transition equation describes the dynamics of the vector of latent state variables, with the linear equation linking the observed yields with state vector. In contrast to F. Diebold, G. Rudebusch and B. Aruoba (6), this study makes use of independent autoregressive first order specifications for each state variable, i.e. all non-diagonal elements of matrix A are equal to zero. This approach allows for a considerable reduction in the number of coefficients to be estimated, taking into account the condition of a rather short time series. In the research of F. Diebold, G. Rudebusch and B. Aruoba, all off-diagonal coefficients are insignificant therefore it justifies the specification selection for the present study. In addition, the work of F. Diebold and C. Li (4) deals with the autoregressive specification of L_t, C_t, S_t as well, yet the estimation of the coefficients, instead of being carried out using the Kalman filter (as in the present study due to its advantages), is based on the two-step method.

Several model parameters are to be estimated. The (3 x 3) dimensional transition diagonal matrix A comprises three free parameters, the (3 x 1) dimensional constant vector μ has three free parameters, and the measurement matrix Λ comprises one free parameter λ. In addition, the transition and disturbance covariance matrix Q contains three free parameters (one disturbance variance for each of the three latent factors: level, slope and curvature), while the measurement disturbance covariance matrix H has five free parameters (one disturbance variance for each of the five yields). Consequently, overall 15 parameters are to be estimated via optimisation, and it is a complex numerical exercise.

For computing optimal yield forecasts and the related errors, the Kalman filter has been applied to this configuration of parameters; afterwards, the Gaussian likelihood function has been estimated for the model using the prediction-error decomposition of the likelihood. The theoretical frame of the Kalman filter is detailed in Appendix 1. For initialisation of the Kalman filter, the values of the state variables are used, applying the least square method and cross-sectional data at time $t = 1$ (the first observation in the time series).

Table 2.2.1 presents the results of the estimated model. Autoregressive coefficients L_t, C_t and S_t point to a highly persistent dynamics (0.99, 0.88 and 0.85, respectively). The values thus obtained are extremely close to those in F. Diebold, G. Rudebusch and

[3] As of May 2000, the Bank of Latvia started the computation of 9- (for internal use) and 12-month RIGIBID and RIGIBOR, therefore, for the purpose of a larger sample, the period starting with May 2000 has been selected.

B. Aruoba (the coefficients are 0.99, 0.94 and 0.84, respectively). As in the study of the above authors, when moving from L_t to S_t to C_t the transition shock volatility increases. Though all the constants are insignificant, they remain in the specification of this paper to rule out the possibility of unstable estimates.

Table 2.2.1
Model Parameter Estimates

L	S	C	μ_L	μ_S	μ_C	σ_L	σ_S	σ_C
0.994	0.890	0.850	–0.054	–0.078	–0.002	0.011	0.132	0.271
(0.013)	(0.046)	(0.098)	(0.087)	(0.091)	(0.121)	(0.015)	(0.021)	(0.077)

Note. Standard errors of coefficients are given in brackets.

2.3 The Risk Premium

In the study, the risk premium is defined as the difference between the forward rate f and the expected future interest rate of the respective maturity $E_t i(\tau,T)$:

$$\mathrm{pr}(t,\tau,T) = f(t,\tau,T) - E_t i(\tau,T)$$

where $E_t i(\tau,T)$ is derived using

$$E_t i(\tau,T) = L_{\tau|t} + S_{\tau|t}\left(\frac{1-e^{-\lambda(T-\tau)}}{\lambda(T-\tau)}\right) + C_{\tau|t}\left(\frac{1-e^{-\lambda(T-\tau)}}{\lambda(T-\tau)} - e^{-\lambda(T-\tau)}\right)$$

where $(L_{\tau|t}, S_{\tau|t}, C_{\tau|t}) = \alpha'_{\tau|t} \equiv E_t(\alpha'_\tau)$ is the forecast of state variables $(\tau - t)$ steps ahead under the condition that the initial value of $\alpha_{t|t}$ is the state variable values filtered at time t.

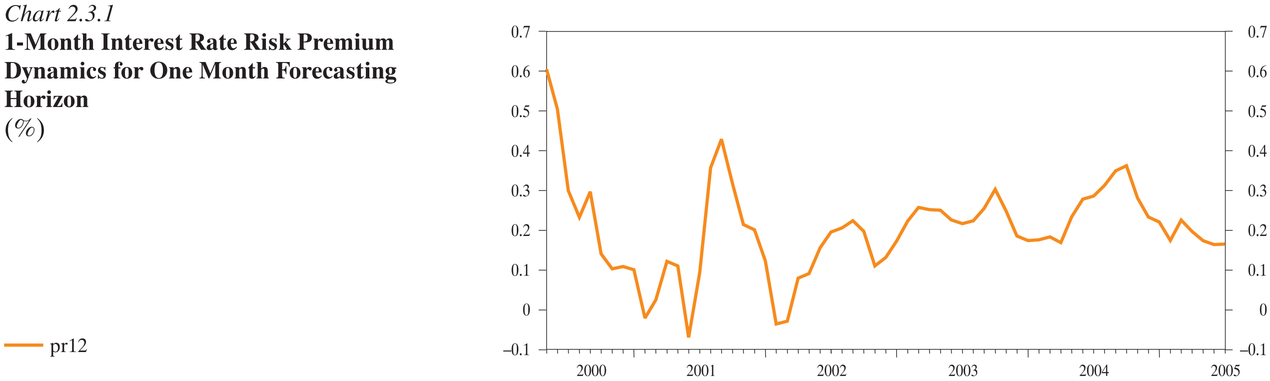
The filtered values of state variables are used also in the calculations of forward rates:

$$f(t,\tau,T) = \frac{i^*(t,T)\cdot(T-t) - i^*(t,\tau)\cdot(\tau-t)}{(T-\tau)\cdot\left(1+i^*(t,\tau)\cdot(\tau-t)\right)}$$

where $i^*(t,\tau) = L_{t|t} + S_{t|t}\left(\frac{1-e^{-\lambda(\tau-t)}}{\lambda(\tau-t)}\right) + C_{t|t}\left(\frac{1-e^{-\lambda(\tau-t)}}{\lambda(\tau-t)} - e^{-\lambda(\tau-t)}\right)$ is the theoretical interest rate value consistent with the Nelson–Siegel model at time *t.*

Chart 2.3.1 shows risk premium pr(t,1,2) on 1-month interest rate for one month forecasting horizon. It should be noted that the risk premium defined in this study is correct, if the estimated expectation measure is correct as a forecast obtained with the help of the Kalman filter. Chart 2.3.1 demonstrates that up to 2002, the volatility of risk premium was substantial, then it stabilised, and after that its decline from 36 basis points in October 2004 to 16 basis points in July 2005 was recorded. At the close of 2004, the said decline in risk premium was associated with the market participants' expectations for repegging of the lats from the SDR basket of currencies to the euro, which implied a smaller exchange risk. The risk premium continued to shrink also at the beginning of 2005, which was likely to be associated with the ongoing convergence. Appendix 2 shows the dynamics of 1-month interest rate risk premium for the horizon of 2–12 months. Some points in time with negative risk premiums notwithstanding, the premium remains positive on average over the entire reporting period. Negative risk premiums may imply the presence of a number of expectation errors. The premium values for forecasting horizons exceeding 4 months are only positive. Appendices 3, 4 and 5 highlight a similar behaviour for 3-, 6- and 12-month interest rate premiums.

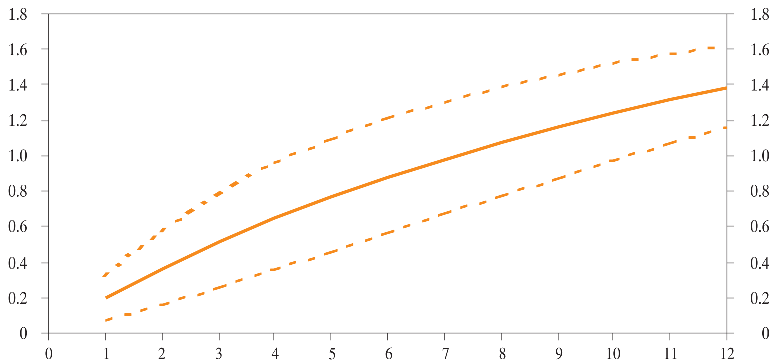
Chart 2.3.1
1-Month Interest Rate Risk Premium Dynamics for One Month Forecasting Horizon
(%)



For the purpose of estimating the average premium of the reviewed period, the arithmetic mean premium of the period and its standard deviation for 1-, 3-, 6- and 12-month interest rates have been computed.

Chart 2.3.2 and 2.3.3 show arithmetic mean risk premiums and their standard deviations for 1–12 month forecasting horizon. The premiums are positive, according to the prediction, and, with the forecasting horizon growing, they increase. The longer the horizon, the larger premium is demanded by market participants. By contrast, standard errors display a tendency to increase in the period up to six month horizon, after which they decline steadily. The tables of Appendix 6 report mean risk premiums on 3-, 6- and 12-month interest rates and their standard deviations. Data in Appendix 6 lead to an inference that the behaviour of 3-, 6- and 12-month interest rate premiums and standard deviations is similar to the behaviour of 1-month interest rates.

Chart 2.3.2
1-Month Interest Rate Period Average Risk Premium for Different Forecasting Horizons
(risk premium within the interval of ± one standard deviation for 1–12 month forecasting horizon; in months; %)



Premiums with the same forecasting horizon but different term to maturity display a peculiar tendency of behaviour. With the interest rate term increasing, the risk premium decreases. Chart 2.3.4 demonstrates the behaviour of 1-, 3-, 6- and 12-month risk premiums for one month forecasting horizon. The Chart shows that over a longer term the risk premium decreases. A similar relation is observed also for 2–12 month forecasting horizon. The theoretical reasoning for the behaviour of the risk premium within this model is provided in Chapter 2.4.

Chart 2.3.3
1-Month Interest Rate Period Average Standard Deviation for Different Forecasting Horizons
(in months; %)

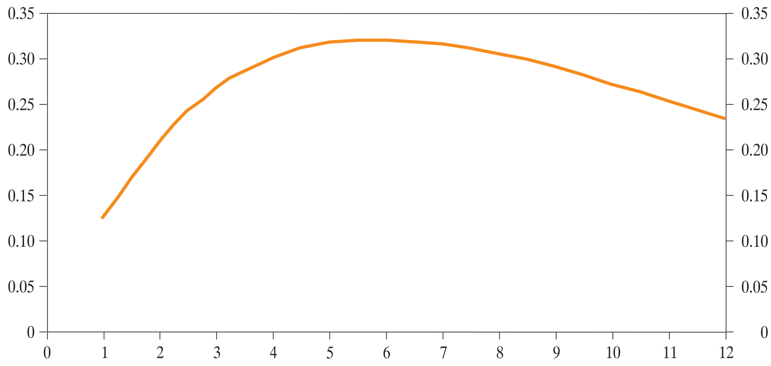


Chart 2.3.4
1-, 3-, 6- and 12-Month Interest Rate Risk Premium for One Month Forecasting Horizon
(in months; %)

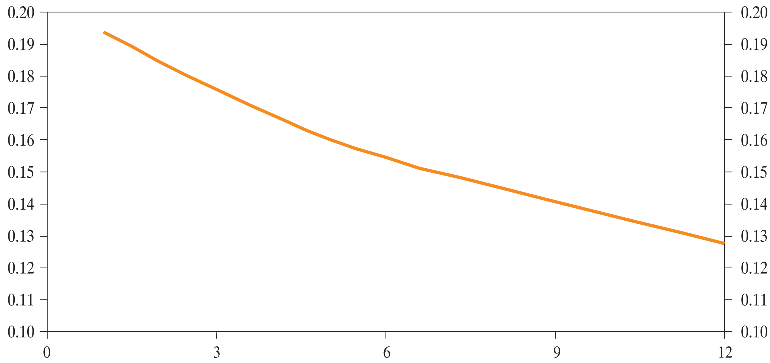


Table 2.3.1 presents the comparison of Latvian interest rate risk premiums with those of other countries and their evaluation in different sources. The analysis of the data leads to a conclusion that at present the interest rate risk premiums in Latvia are higher than in the developed countries. The ongoing convergence is likely to bring interest rate risk premiums in lats closer to those of the euro area.

Table 2.3.1
Risk Premiums in Latvia and Other Countries
(1-month interest rate risk premiums for different forecasting horizons; in basis points)

	1 month	3 months	6 months	9 months
Latvia[1]	19	52	88	116
1-month LIBOR Germany (December 1989–December 1998)[2]	5	10	15	27
EONIA swap rates				
January 1999–September 2001[2]	0	2	6	10
January 1999–June 2002[2]	1	2	4	13
Germany (1972–1998)[3]	0–5	5–10	20–25	25–35
Canada (1988–1998)[4]	6	29	58	100

[1] Results of this paper.
[2] (8).
[3] (2).
[4] (13).

2.4 Explanation of Risk Premium Behaviour

This Chapter attempts to explain theoretically the empirical behaviour of the above-referred risk premium within the framework of the researched model.

In compliance with equation [1.2] above

$$\mathrm{pr}(t,\tau,T) = f(t,\tau,T) - E_t i(\tau,T) \quad [2.1].$$

For simplicity, we shall replace equation [1.1] with the following definition of the forward rate:

$$f(t,\tau,T) = \frac{i(t,T)\cdot(T-t) - i(t,\tau)\cdot(\tau-t)}{(T-\tau)} \quad [2.2],$$

which corresponds to a continuously compounded rate.

The substitution of elements from equation [1.4] into [2.2] instead of $i(t,\ \tau)$ and $i(t,\ T)$ gives

$$f(t,\tau,T) = L_t + S_t \frac{e^{-\lambda(\tau-t)} - e^{-\lambda(T-t)}}{\lambda(T-\tau)} + C_t\left[\frac{e^{-\lambda(\tau-t)} - e^{-\lambda(T-t)}}{\lambda(T-\tau)} + (\tau-t)e^{-\lambda(\tau-t)} - (T-t)e^{-\lambda(T-t)}\right] \quad [2.3].$$

The interest rate forecast from equation [1.4] is

$$E_t i(\tau,T) = E_t(L_\tau) + \frac{1-e^{-\lambda(T-\tau)}}{\lambda(T-\tau)} E_t(S_\tau) + \left(\frac{1-e^{-\lambda(T-\tau)}}{\lambda(T-\tau)} - e^{-\lambda(T-\tau)}\right) E_t(C_\tau) \quad [2.4]$$

where $E_t(L_\tau)$, $E_t(S_\tau)$ and $E_t(C_\tau)$ are forecasts of the respective factors at time t for the future period τ.

Accounting for the factor dynamics' subjection to the first order autoregressive process, let us produce the following factor forecast equations:

$$E_t(L_\tau) = a_{11}^{\tau-t} L_t + \mu_L\left(1 + a_{11} + \ldots + a_{11}^{\tau-t-1}\right) = a_{11}^{\tau-t} L_t + \mu_L \frac{1-a_{11}^{\tau-t}}{1-a_{11}} \quad [2.5],$$

$$E_t(S_\tau) = a_{22}^{\tau-t} S_t + \mu_S\left(1 + a_{22} + \ldots + a_{22}^{\tau-t-1}\right) = a_{22}^{\tau-t} S_t + \mu_S \frac{1-a_{22}^{\tau-t}}{1-a_{22}} \quad [2.6],$$

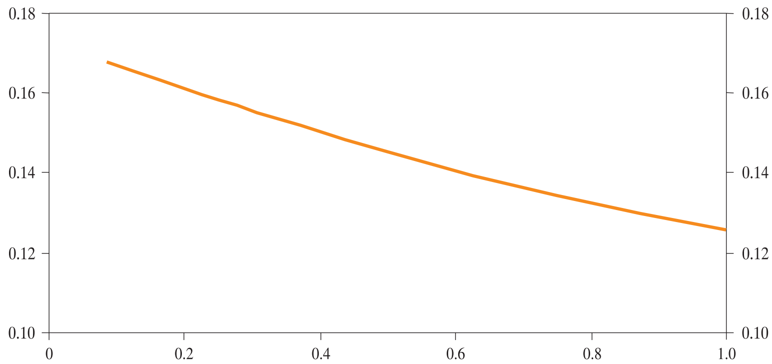
$$E_t(S_\tau) = a_{33}^{\tau-t} C_t + \mu_C \frac{1-a_{33}^{\tau-t}}{1-a_{33}} \quad [2.7].$$

Equations [2.1] and [2.3]–[2.7] give the following risk premium:

$$\operatorname{pr}(t,\tau,T) = f(t,\tau,T) - E_t i(\tau,T) = L_t + S_t \frac{e^{-\lambda(\tau-t)} - e^{-\lambda(T-t)}}{\lambda(T-\tau)} +$$
$$+ C_t \left[\frac{e^{-\lambda(\tau-t)} - e^{-\lambda(T-t)}}{\lambda(T-\tau)} + (\tau - t) e^{-\lambda(\tau-t)} - (T-t) e^{-\lambda(T-t)} \right] - a_{11}^{\tau-t} L_t - \mu_L \frac{1-a_{11}^{\tau-t}}{1-a_{11}} -$$
$$- \left(\frac{1-e^{-\lambda(T-\tau)}}{\lambda(T-\tau)} \right) \left(a_{22}^{\tau-t} S_t + \mu_S \frac{1-a_{22}^{\tau-t}}{1-a_{22}} \right) - \left(\frac{1-e^{-\lambda(T-\tau)}}{\lambda(T-\tau)} - e^{-\lambda(T-\tau)} \right) \left(a_{33}^{\tau-t} C_t + \mu_C \frac{1-a_{33}^{\tau-t}}{1-a_{33}} \right)$$
[2.8].

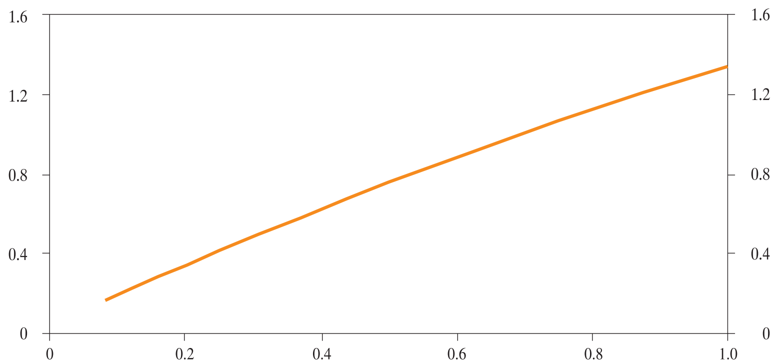
Chart 2.4.1 shows the risk premium from equation [2.8] as the function of interest rate maturity ($T - t$) at the forecasting horizon $\tau - t = 1$ month. Factor values of this risk premium $L_t = 2.756$, $S_t = -0.482$, $C_t = 0.048$, and coefficients a_{ii} and μ_j correspond to the Kalman filter estimates of July 2005, i.e. the last sample observation. The Chart discloses that at the given parameter values the function is declining, if $T - t$ is growing, and this is in support of the empirical facts referred to in the previous Chapter.

Chart 2.4.1
Theoretical Interest Rate Risk Premium for One Month Forecasting Horizon
(risk premium as a function from interest rate maturity; in years; %)



When we fix the interest rate term $T - \tau = 1$ month for equation [2.8] with these parameter values and change the forecasting horizon, we obtain the risk premium curve, which, depending on the horizon, is presented in Chart 2.4.2. The Chart shows that when the forecasting horizon increases the risk premium grows. It confirms the empirical facts and heuristic assumptions from Chapter 2.3 about investors demanding higher risk premiums for larger uncertainty related to investments at a more distant point in time.

Chart 2.4.2
Theoretical 1-Month Interest Rate Risk Premium for Different Forecasting Horizons
(risk premium as a function from interest rate maturity; in years; %)



CONCLUSION

The paper presents the analysis of risk premium of the interest rate term structure for the Latvian money market. The risk premium has been defined as the difference between the forward interest rate and the expected future interest rate of the respective maturity. The interest rate term structure is assessed consistently with the Nelson–Siegel model.

On the back of the approach used by F. Diebold, G. Rudebusch and B. Aruoba, it has been assumed that the coefficients of the Nelson–Siegel model are unobservable, therefore the model of this research paper has been estimated using the Kalman filter.

RIGIBID and RIGIBOR interest rates of the Latvian money market have been used as the observable variables.

The expected future interest rate has been computed as the Kalman filter forecast n periods ahead.

The risk premium behaviour has been obtained for interest rates of different maturities and forecasting horizons between May 2000 and July 2005. The results obtained indicate that the amount of the risk premium was significant and its volatility substantial between 2000 and 2002. In post-2002 period, its behaviour gradually stabilised and was marked by a downward trend after 2004.

This is in support of the assumption that with the forecasting horizon increasing the risk premium grows, while with the expansion of maturity it becomes smaller. These facts have been explained theoretically in the paper on the basis of the model's mathematical structure.

The risk premium estimation becomes more complicated when the market participants' expectations embedded in the financial market data are to be analysed. The respective analysis carried out within this research has proposed an additional instrument for a CB to timely identify the market participants' expectations regarding interest rates in the future and to learn about their confidence in the conducted monetary policy. Nevertheless, the employment of such instruments as money market RIGIBID and RIGIBOR restricts the forecasting horizon to one year.

In order to expand the forecasting horizon and to enhance the accuracy of the risk premium estimates, the up-coming research foresees to include in the model the bond market data, which are better determinants of the convergence process (to euro interest rates). In such circumstances, the application of the Kalman filter has particular advantages, for rare transactions and irregular quotations are characteristic for the government bond market of Latvia.(3) In addition, bond market data are non-stationary. The Kalman filter has another positive trait: it implicitly allows for accounting of such unobservable factors as investment and political climate, whose correct quantitative estimation is impossible.

The including of such macroeconomic variables as inflation, GDP, etc (6) in the model opens up another potential area of investigation. It would elucidate whether the interest rate term structure carries information that would allow for forecasting macroeconomic variables, and, vice versa, whether macroeconomic variables allow for improving interest rate forecasts. The results regarding risk premium behaviour obtained by such a method would be comparable with the findings of the factor models used in the current study; it would likewise be possible to assess the accuracy of the computed risk premium.

APPENDICES

Appendix 1
Detailed Description of the Kalman Filter Methodology

A.1 Properties of conditional mathematical expectations

x and y denote random vectors whose joint distribution has the first and the second moments. The second moments are defined as

$$\begin{cases} D(x) = E(x \cdot x') - E(x) \cdot E(x') \\ D(y) = E(y \cdot y') - E(y) \cdot E(y') \\ C(y, x) = E(y \cdot x') - E(y) \cdot E(x') \end{cases} \qquad \{1\}$$

where ' denotes the transposed matrix.

It is assumed that the conditional expected y value with condition x (which holds at a normal joint distribution of x and y) can be written as a linear function

$$E(y \mid x) = \alpha + B' x \qquad \{2\}.$$

Vector α and matrix B' will be expressed as moments of equation system {1}. Using the property of conditional mathematical expectations

$$E\{E(y \mid x)\} = E(y) \qquad \{3\},$$

from equation {2} we obtain

$$E(y) = \alpha + B' E(x) \qquad \{4\}$$

or

$$\alpha = E(y) - B' E(x) \qquad \{5\}.$$

Multiplying equation {2} by x' gives

$$E(y \mid x) \cdot x' = \alpha \cdot x' + B' x \cdot x'.$$

When computing mathematical expectations for the right and left side of the equation and using equation {3}, we obtain

$$E\{E(y \mid x) \cdot x'\} = E\{E(y \cdot x' \mid x)\} = E(y \cdot x') = \alpha \cdot E(x') + B'[E(x \cdot x')]$$

or

$$E(y \cdot x') = \alpha \cdot E(x') + B'[E(x \cdot x')] \qquad \{6\}.$$

Multiplying equation {4} by E(x'), we obtain

$$E(y) \cdot E(x') = \alpha \cdot E(x') + B'E(x) \cdot E(x') \qquad \{7\}.$$

Subtracting equation {6} from equation {7} and using system {1}, we obtain

$C(y,x) = E(y \cdot x') - E(y) \cdot E(x') = B'(E(x \cdot x') - E(x) \cdot E(x')) = B'D(x)$ {8},

thus arriving at

$B' = C(y,x)D^{-1}(x)$ {9}.

Substituting B' from equation {9} and α from equation {5} into equation {2}, we obtain

$E(y \mid x) = \alpha + B'x = E(y) - B'E(x) + B'x =$

$= E(y) - B'(x - E(x)) = E(y) - C(y,x)\, D^{-1}(x)(x - E(x))$

or

$E(y \mid x) = E(y) - C(y,x)\, D^{-1}(x)(x - E(x))$ {10}.

The following representation is derived in a similar way:

$D(y \mid x) = D(y) - C(y,x)\, D^{-1}(x) \cdot C(x,y)$ {11}.

A.2 The Kalman filter

The representation of (n x 1) state-space dynamics of dimensional vector y_t can be defined with the following equation system:

$y_t = c_t + Z_t\alpha_t + \varepsilon_t$ {12},

$\alpha_{t+1} = d_t + T_t\alpha_t + v_{t+1}$ {13}

where
α_t is the (m x 1) dimensional vector of unobservable variables;
c_t, d_t, Z_t, T_t are vectors and matrices of respective dimensions;
ε_t and v_t are Gaussian random vectors with zero mean.

Equation {12} is often referred to as the signal or observation equation, while equation {13} is known as the state or transition equation.

Random vectors ε_t and v_t are treated as serially uncorrelated in time, with the following covariance matrix:

$$\Omega_t = \mathrm{var}\begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} = \begin{bmatrix} H_t & G_t \\ G'_t & Q_t \end{bmatrix} \qquad \{14\}.$$

It is assumed that vectors ε_t and v_t are white noises vectors, i.e.

$$E(\varepsilon_t \cdot \varepsilon'_\tau) = \begin{cases} H, t = \tau \\ 0, t \neq \tau \end{cases}, \quad E(\nu_t \cdot \nu'_\tau) = \begin{cases} Q, t = \tau \\ 0, t \neq \tau \end{cases} \qquad \{15\}$$

where H and Q are symmetrical matrices of (n x n) and (m x m) dimensions, respectively.

It is also assumed that ε_t and v_t do not correlate for all lags:

$$E(\varepsilon_t \cdot v'_\tau) = 0 \quad \forall \ t, \tau \qquad \{16\}.$$

The Kalman filter is applied for an optimal estimation of vector α_t of unobservable variables and for estimation updating when new values of observable variables become available. Optimal forecasts for endogenous variable y_t are acquired at the same time.

We assume the need to calculate $\alpha_{t|t}$ – the optimal estimate (with minimum mean squared error) α_t using information available up to time t, and $\Omega_{t|t}$, which is the error covariance matrix for the forecast in state equations. It is also assumed that vectors c and d as well as matrices Z and T are known.

The recurrent algorithm of the Kalman filter includes the following steps.

1. Selection of the initial state.
$\alpha_{1|0}$ denotes the predicted value of α_1, which is based on the initial value of y_0. If all eigenvalues of matrix T are smaller than 1 by their absolute values, it is assumed that $\alpha_{1|0} = E(\alpha_1)$, i.e. unconditional mean value of the process.

We assume that $\Omega_{1|0}$ satisfies the equation

$$\Omega_{1|0} = T \cdot \Omega_{1|0} T' + Q \qquad \{17\},$$

which is consistent with unconditional covariance matrix of the process.

If some eigenvalues of matrix T exceed or are equal to unit, the unconditional mean of the process and covariance cannot be selected as initial values (as not existing), and hence the selection shall be made on the back of other considerations.

When the initial values $\alpha_{1|0}$ and $\Omega_{1|0}$ are known, the next action is to calculate $\alpha_{2|1}$ and $\Omega_{2|1}$ for the next time moment. As all computations for periods t = 2, 3, ..., T are analogous, the transition computation algorithm for any period t from values $\alpha_{t|t-1}$, $\Omega_{t|t-1}$ to values $\alpha_{t+1|t}$, $\Omega_{t+1|t}$ is used.

2. Y_t predicting and construction of its covariance matrix

$$\hat{y}_{t|t-1} = c + Z\alpha_{t|t-1} \qquad \{18\},$$

$$E(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})' = E(Z(\alpha_t - \alpha_{t|t-1})(\alpha_t - \alpha_{t|t-1})Z' + H =$$
$$= Z\Omega_{t|t-1}Z' + H = \sum\nolimits_{t|t-1} \qquad \{19\}.$$

3. Assuming that y_t value becomes available at time t. This information allows for the adjustment of forecast $\alpha_t - \alpha_{t|t-1}$.

Y_{t-1} denotes vector $(y_0, y_1, ..., y_{t-1})'$. Thus from equation {12} we obtain

$$
\begin{aligned}
&\operatorname{cov}(\alpha_t, y_t | Y_{t-1}) = \operatorname{cov}(\alpha_t, Z \cdot \alpha_t | Y_{t-1}) = \\
&= E((\alpha_t - \alpha_{t|t-1})(\alpha_t - \alpha_{t|t-1})' Z' | Y_{t-1}) = \Omega_{t|t-1} Z', \\
&D(y_t | Y_{t-1}) = D(c + Z\alpha_t + \varepsilon_t | Y_{t-1}) = Z' \Omega_{t|t-1} Z + H, \\
&E(\alpha_t | Y_{t-1}) = \alpha_{t|t-1}.
\end{aligned}
$$

Using equation {10} (substituting y with α_t, x with y_t, and replacing the unconditional mathematical expectation with equation $E(\cdot \mid Y_{t-1})$), we obtain the adjusted value of $\alpha_{t|t}$:

$$
\hat{\alpha}_{t|t} = \hat{\alpha}_{t|t-1} + \{E[(\alpha_t - \hat{\alpha}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\} \cdot \{E[(y_t - \hat{y}_{t|t-1})(y_t - \hat{y}_{t|t-1})']\}^{-1} (y_t - \hat{y}_{t|t-1}) \qquad \{20\},
$$

while

$$
\begin{aligned}
&E[(\alpha_t - \hat{\alpha}_{t|t-1})(y_t - \hat{y}_{t|t-1})'] = E[(\alpha_t - \hat{\alpha}_{t|t-1})((\alpha_t - \alpha_{t|t-1})' Z' + \varepsilon_t)] = \\
&= E[(\alpha_t - \hat{\alpha}_{t|t-1})(\alpha_t - \alpha_{t|t-1})'] Z' = \Omega_{t|t-1} \cdot Z' \qquad \{21\}.
\end{aligned}
$$

The condition that ε_t is uncorrelated to other factors is used.
Substituting equation {19} into equation {20}, we obtain

$$
\hat{\alpha}_{t|t} = \hat{\alpha}_{t|t-1} + \Omega_{t|t-1} \cdot Z' (Z \cdot \Omega_{t|t-1} Z' + H)^{-1} (y_t - c - Z\alpha_{t|t-1}) \qquad \{22\}.
$$

Covariance matrix for errors related to the given adjusted forecast is obtained from equation {11}:

$$
\begin{aligned}
&\Omega_{t|t} = E[(\alpha_t - \hat{\alpha}_{t|t})(\alpha_t - \hat{\alpha}_{t|t})'] = \\
&= E[(\alpha_t - \hat{\alpha}_{t|t-1})(\alpha_t - \hat{\alpha}_{t|t-1})'] - \{E[(\alpha_t - \alpha_{t|t-1})(y_t - y_{t|t-1})']\} \cdot \\
&\cdot \{E[(y_t - y_{t|t-1})(y_t - y_{t|t-1})']\}^{-1} \cdot \{E[(y_t - y_{t|t-1})(\alpha_t - \alpha_{t|t-1})']\} = \\
&= \Omega_{t|t-1} - \Omega_{t|t-1} Z' (Z \cdot \Omega_{t|t-1} \cdot Z' + H)^{-1} Z \cdot \Omega'_{t|t-1} \qquad \{23\}.
\end{aligned}
$$

The difference between the adjusted value $\alpha_{t|t}$ and value $\alpha_{t|t-1}$, which was predicted before information about y_t became available, is presented as $\Omega_{t|t-1} \cdot Z'(Z \cdot \Omega_{t|t-1} Z' + H)^{-1}(y_t - c - Z\alpha_{t|t-1})$. Hence the larger the value of the expression $y_t - c - Z\alpha_{t|t-1} = y_t - y_{t|t-1}$, i.e. the difference between the realised and predicted value of y_t, the larger the adjustment ($\alpha_{t|t} - \alpha_{t|t-1}$); however, the given value is inversely proportional to the forecasting accuracy, which is consistent with $\sum_{t|t-1} = Z\Omega_{t|t-1} Z' + H$, and directly proportional to covariance between α_t and $y_t - \Omega_{t|t-1} Z'$. Consequently, the less accurate the forecast $y_{t|t-1}$, the smaller the value of the adjustment term in equation {23}, and the larger the conditional covariance between α_t and y_t, the larger the adjustment term.

4. Derivation of the state variable forecast for the next period from equation {13}:

$$\alpha_{t+1|t} = E(\alpha_{t+1}|Y_t) = E(d + T\cdot\alpha_t + v_{t+1}|Y_t) = d + T\cdot E(\alpha_t|Y_t) + E(v_{t+1}|Y_t) = d + T\cdot\alpha_{t|t} \qquad \{24\}.$$

Substituting equation {22} into equation {24}, we obtain

$$\alpha_{t+1|t} = d + T\alpha_{t|t-1} + T\Omega_{t|t-1}\cdot Z'(Z\cdot\Omega_{t|t-1}\cdot Z' + H)^{-1}(y_t - c - Z\alpha_{t|t-1}) \qquad \{25\}.$$

Matrix

$$k_t = T\Omega_{t|t-1}\cdot Z'(Z\cdot\Omega_{t|t-1}\cdot Z' + H)^{-1} \qquad \{26\}$$

is known as the gain matrix, and equation {25} can be rewritten

$$\alpha_{t+1|t} = d + T\alpha_{t|t-1} + k\cdot(y_t - c - Z\alpha_{t|t-1}) \qquad \{27\}.$$

The measurement error covariance matrix for this forecast can be computed using equations {13} and {24}:

$$\begin{aligned}\alpha_{t+1|t} &= E[(\alpha_{t+1} - \alpha_{t+1|t})(\alpha_{t+1} - \alpha_{t+1|t})'] = \\ &= E[(d + T\cdot\alpha_t + v_{t+1} - d - T\cdot\alpha_{t|t})(d + T\cdot\alpha_t + v_{t+1} - d - T\cdot\alpha_{t|t})'] = \\ &= T\cdot E[(\alpha_t - \alpha_{t|t})(\alpha_t - \alpha_{t|t})']T' + E(v_{t+1}v'_{t+1}) = T\cdot\Omega_{t|t}\cdot T' + Q_t \qquad \{28\}.\end{aligned}$$

Substitution of equation {23} into equation {28} results in

$$\Omega_{t+1|t} = T\cdot[\Omega_{t|t-1} - \Omega_{t|t-1}\cdot Z'(Z\cdot\Omega_{t|t-1}\cdot Z' + H)^{-1}Z\cdot\Omega'_{t|t-1}]\cdot T' + Q_t \qquad \{29\}.$$

A.3 Applying the Kalman filter to forecasts *n* periods ahead

Via recursive substitution, equation {13} produces

$$\alpha_{t+n} = T^n\alpha_t + T^{n-1}v_{t+1} + T^{n-2}v_{t+2} + \ldots + T^1v_{t+n-1} + v_{t+n}\text{, where } n = 1, 2, 3\ldots \qquad \{30\}.$$

Projecting α_{t+n} to α_t and Y_t, we obtain

$$E(\alpha_{t+n}|\alpha_t, Y_t) = T^n\alpha_t.$$

The application of conditional mathematical expectations property leads to

$$\alpha_{t+n|t} = E(\alpha_{t+n}|Y_t) = E[E(\alpha_{t+n}|\alpha_t, Y_t)|Y_t] = E(T^n\alpha_t|Y_t) = T^nE(\alpha_t|Y_t) = T^n\alpha_{t|t} \qquad \{31\}.$$

From forecast equations of expressions {30} and {31} *n*-periods ahead, it follows that

$$\alpha_{t+n} - \alpha_{t+n|t} = T^n(\alpha_t - \alpha_{t|t}) + T^{n-1}v_{t+1} + T^{n-2}v_{t+2} + \ldots + T^1 v_{t+n-1} + v_{t+n} \qquad \{32\}.$$

The measurement error covariance matrix is

$$\Omega_{t+n|t} = T^n \cdot \Omega_{t|t}(T')^n + T^{n-1}Q(T')^{n-1} + T^{n-2}Q(T')^{n-2} + \ldots + TQT' + Q \qquad \{33\}.$$

The following expression for the observed vector is obtained from equation {12}:

$$y_{t+n} = c + Z\alpha_{t+n} + \varepsilon_{t+n}.$$

Consequently, y forecast for n periods ahead can be computed using the relationship

$$y_{t+n|t} = E(y_{t+n}|Y_t) = c + Z\alpha_{t+n|t} \qquad \{34\}.$$

The forecast error, in turn, is characterised by the following relationship:

$$y_{t+n} - y_{t+n|t} = (c + Z\cdot\alpha_{t+n} + \varepsilon_{t+n}) - (c + Z\cdot\alpha_{t+n|t}) = Z(\alpha_{t+n} - \alpha_{t+n|t}) + \varepsilon_{t+n}.$$

The covariance matrix of this error is as follows:

$$E[(y_{t+n} - y_{t+n|t})(y_{t+n} - y_{t+n|t})'] = Z\cdot\Omega_{t+n|t}\cdot Z' + H.$$

A.4 The Kalman filter in model parameter estimation

If the initial state of α_1 and random vectors (ε_t, v_t) are of the Gaussian type, y_t distribution meeting condition Y_{t-1} also is Gaussian with the mean

$$y_{t|t-1} = c + Z\cdot\alpha_{t|t-1}$$

and the measurement error matrix

$$\sum\nolimits_{t|t-1} = Z\cdot\Omega_{t|t-1}\cdot Z' + H.$$

The distribution density is represented by

$$f(y_t|Y_{t-1}) = (2\pi)^{-\frac{n}{2}}\left|Z\cdot\Omega_{t|t-1}\cdot Z' + H\right|^{-\frac{1}{2}}\cdot$$
$$\cdot\exp\{-\frac{1}{2}(y_t - c - Z\cdot\alpha_{t|t-1})'(Z\cdot\Omega_{t|t-1}\cdot Z' + H)^{-1}(y_t - c - Z\cdot\alpha_{t|t+1}) \qquad \{35\}$$

where $t = 1, 2, \ldots, T$.

$f(y_1, \ldots, y_T)$ represents the total vector density $(y_1, \ldots, y_T)$. Taking into account the total density property, it can be represented as follows:

$$f(y_1, ..., y_T) = f(y_T | y_{T-1}, ..., y_1) f(y_{T-1}, ..., y_1) =$$
$$= f(y_T | y_{T-1}, ..., y_1) f(y_{T-1} | y_{T-2}, ..., y_1) = ... = \prod_{j=0}^{T-2} f(y_{T-j} | y_{T-j-1}, ..., y_1) \qquad \{36\}.$$

Taking log of equation $\{33\}$ and using equation $\{32\}$, the following likelihood function is derived:

$$L(y_1, ..., y_T | \varphi) = -\frac{Tn}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log \left| \sum\nolimits_{t|t-1} \right| - \frac{1}{2} \sum_{t=1}^{T} [(y_T - y_{t|t-1}) \sum\nolimits_{t|t-1}^{-1} (y_t - y_{t|t-1})] \qquad \{37\}$$

where

φ is the parameter vector,

$y_t - y_{t|t-1} \sim N(0, \sum_{t|t-1})$, $y_1 \sim N(\bar{y}, \sum_{1|0})$

where $\bar{y}$ is the unconditional mean of the process.

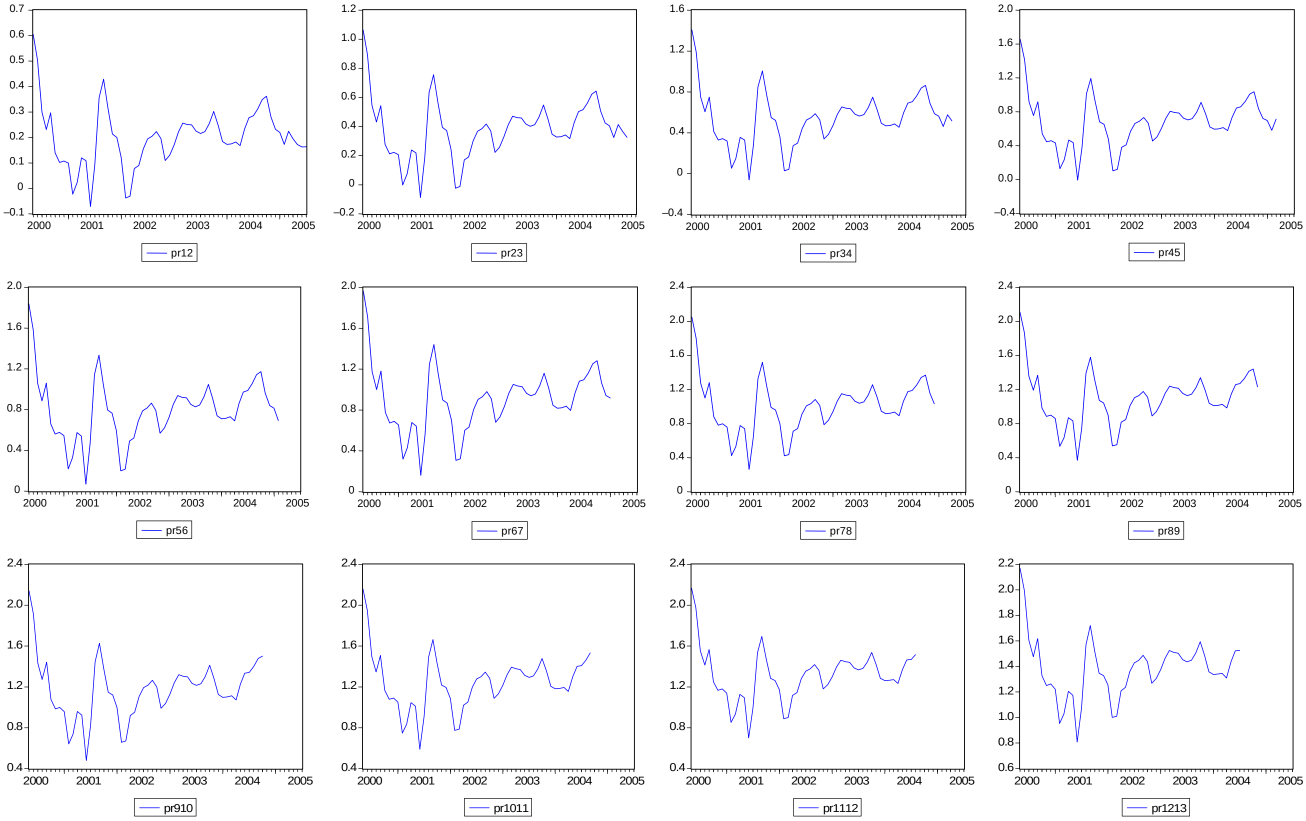
A.5 Model parameter computation algorithm

1. Initial parameter vector φ_0 is selected.
2. Steps 1–4 of the Kalman filter recurrent algorithm are taken (see A.2).
3. For each step, computations are conducted for $y_t - y_{t|t-1}$ and $\sum_{t|t-1}$ that are included in the formation of the likelihood function of equation $\{35\}$.
4. The new value of parameter vector φ_i, which increases L in equation $\{37\}$, is derived by one of the numerical methods.
5. Time steps 2–4 of the Kalman filter recurrent algorithm (see A.2) are repeated until $\left| \varphi^i - \varphi^{i-1} \right| \le \delta$ and $\frac{\partial L(\varphi)}{\partial \varphi} < \delta$ with sufficiently small value δ.

Appendix 2

1-Month Interest Rate Risk Premium Dynamics for Different Forecasting Horizons

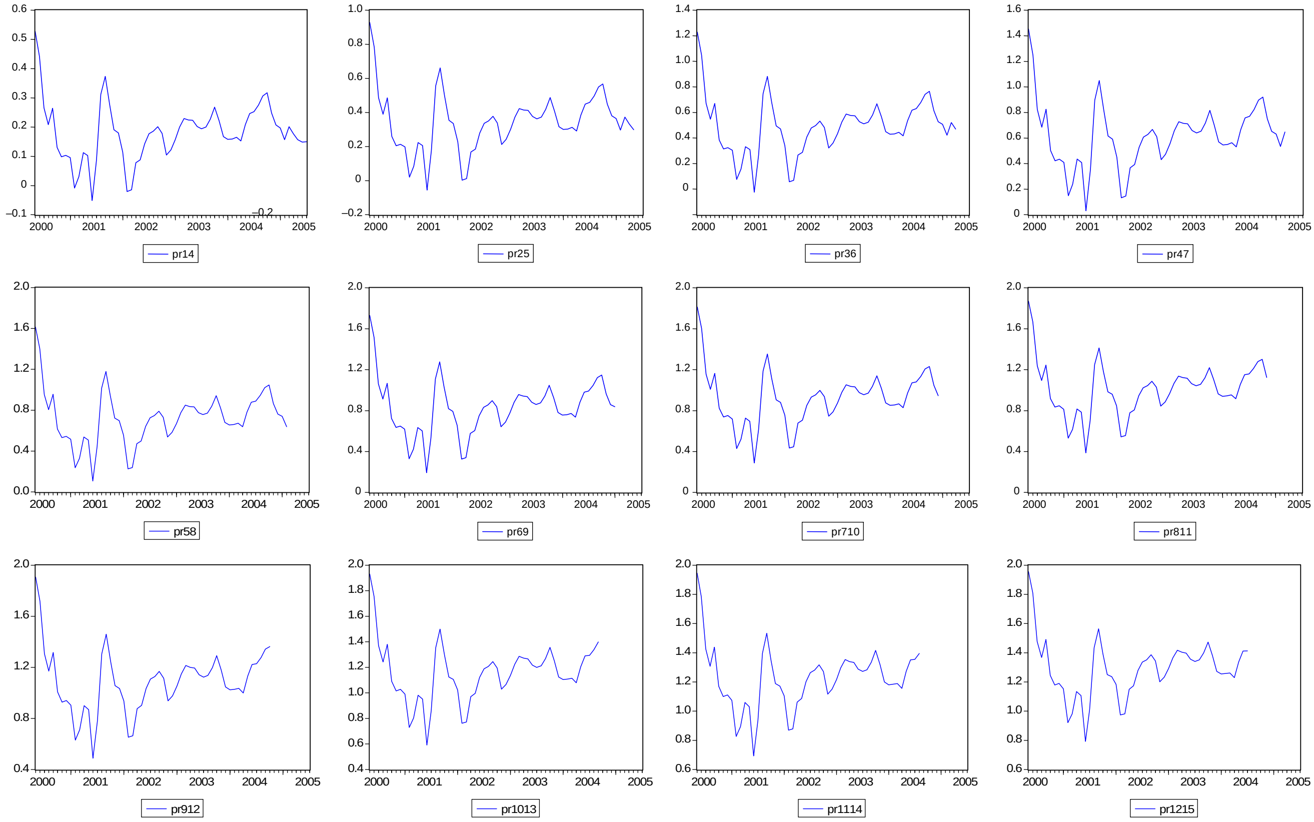
(pr*ij* is premium for forecasting horizon *i* (in months) with repayment period *j* (in months); %)



Appendix 3

3-Month Interest Rate Risk Premium Dynamics for Different Forecasting Horizons

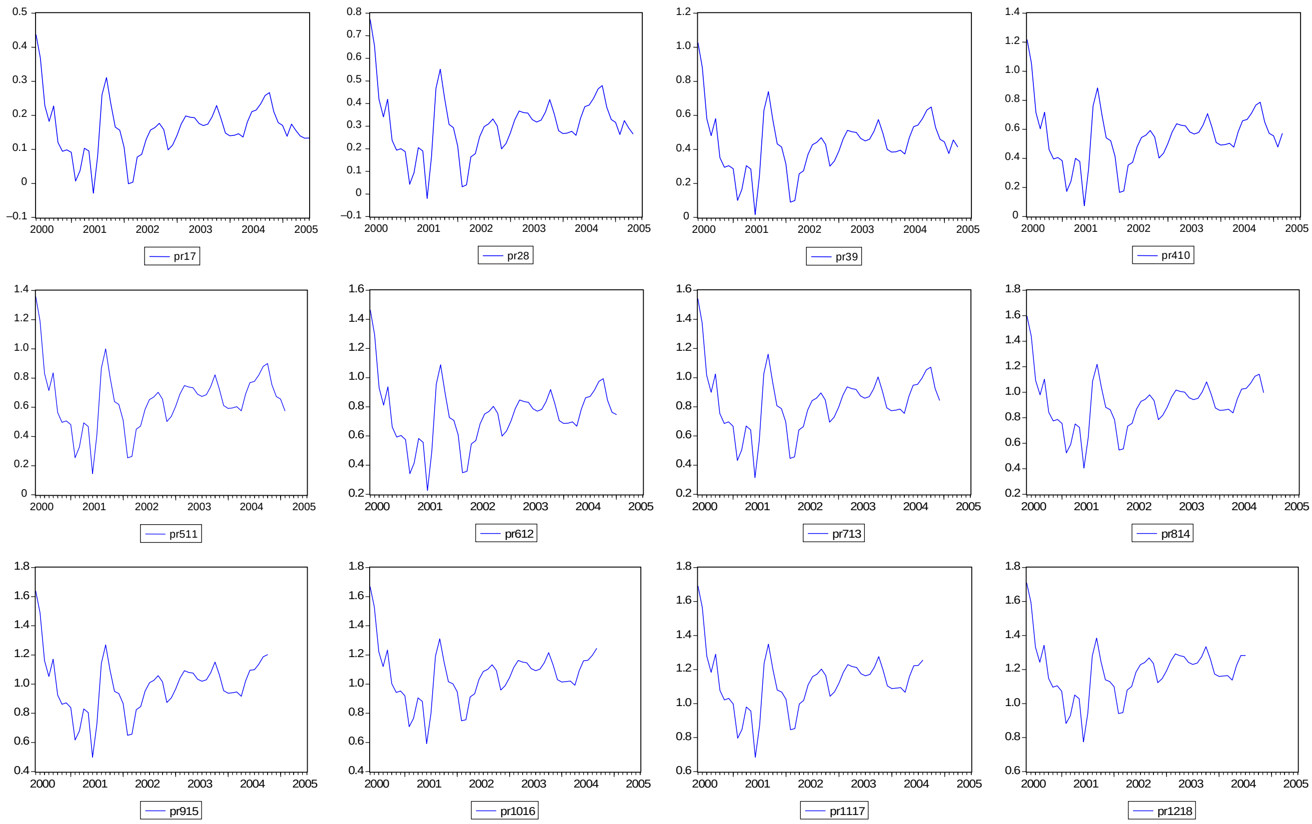
(pr*ij* is premium for forecasting horizon *i* (in months) with repayment period *j* (in months); %)



Appendix 4

6-Month Interest Rate Risk Premium Dynamics for Different Forecasting Horizons

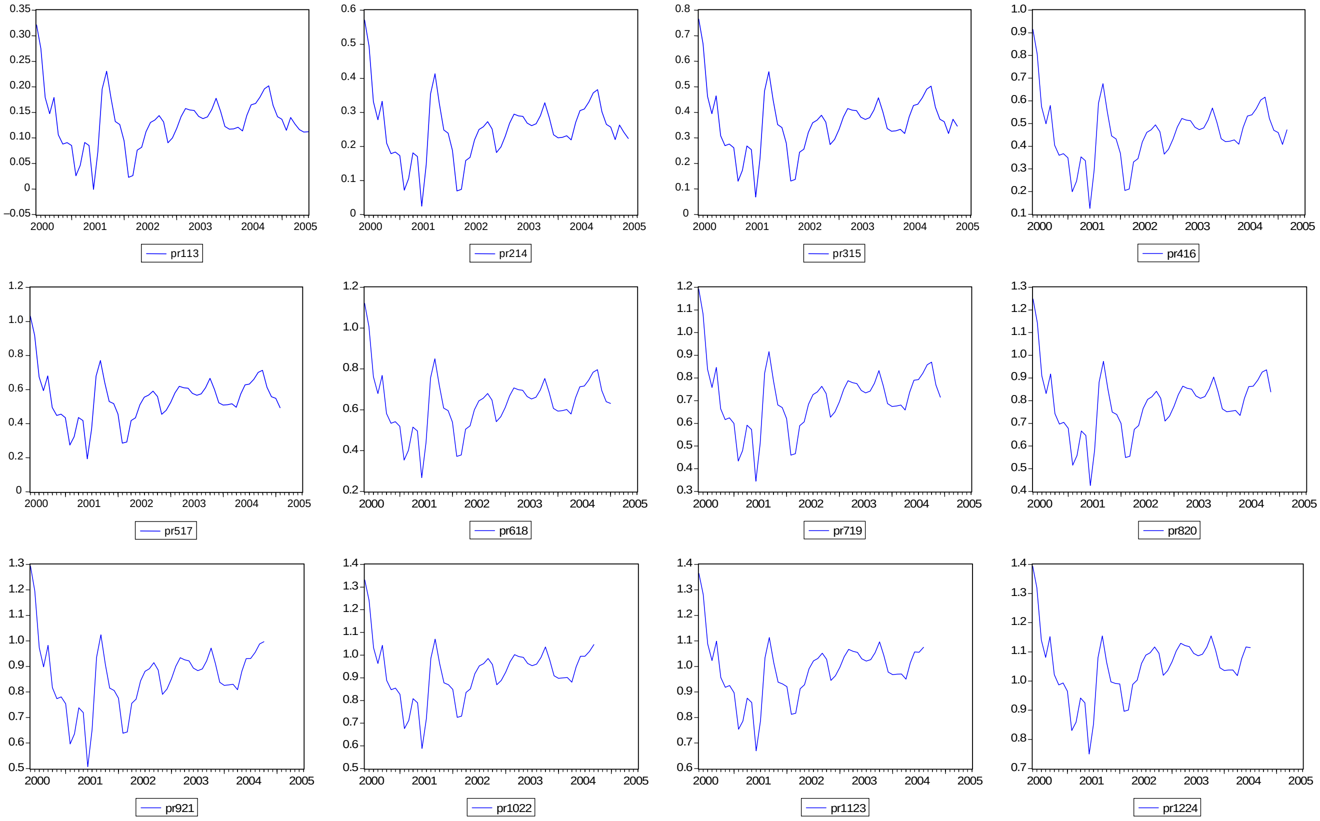
(pr*ij* is premium for forecasting horizon *i* (in months) with repayment period *j* (in months); %)



Appendix 5

12-Month Interest Rate Risk Premium Dynamics for Different Forcasting Horizons

(pr*ij* is premium for forecasting horizon *i* (in months) with repayment period *j* (in months); %)



Appendix 6

Mean Interest Rate Risk Premium and Standard Deviation

(pr*ij* is premium for forecasting horizon *i* (in months) with repayment period *j* (in months); %)

On 1-month interest rate												
	pr12	pr23	pr34	pr45	pr56	pr67	pr78	pr89	pr910	pr1011	pr1112	pr1213
Mean premium	0.193629	0.364347	0.515686	0.650680	0.771919	0.881606	0.981610	1.073508	1.158631	1.238101	1.312857	1.383688
Standard deviation	0.124962	0.211080	0.267350	0.300950	0.317572	0.321701	0.316850	0.305745	0.290489	0.272687	0.253551	0.233989
On 3-month interest rate												
	pr14	pr25	pr36	pr47	pr58	pr69	pr710	pr811	pr912	pr1013	pr1114	pr1215
Mean premium	0.175572	0.331301	0.470243	0.595022	0.707878	0.810715	0.905146	0.992536	1.074034	1.150609	1.223071	1.292104
Standard deviation	0.106270	0.179455	0.227233	0.255724	0.269783	0.273233	0.269068	0.259613	0.246660	0.231579	0.215408	0.198923
On 6-month interest rate												
	pr17	pr28	pr39	pr410	pr511	pr612	pr713	pr814	pr915	pr1016	pr1117	pr1218
Mean premium	0.154646	0.293008	0.417591	0.530540	0.633689	0.728594	0.816575	0.898746	0.976048	1.049272	1.119081	1.186033
Standard deviation	0.084945	0.143384	0.181485	0.204165	0.215318	0.218016	0.214659	0.207116	0.196827	0.184895	0.172159	0.159248
On 12-month interest rate												
	pr113	pr214	pr315	pr416	pr517	pr618	pr719	pr820	pr921	pr1022	pr1123	pr1224
Mean premium	0.127464	0.243277	0.349219	0.446817	0.537372	0.621988	0.701603	0.777008	0.848869	0.917749	0.984119	1.048375
Standard deviation	0.057915	0.097684	0.123556	0.138914	0.146443	0.148255	0.146006	0.140985	0.134193	0.126398	0.118191	0.110020

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